

ALLEGATO B

UNIVERSITÀ DEGLI STUDI DI MILANO

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INFORMAZIONI PERSONALI

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Current position

2017.03.01 – now Postdoctoral Researcher (RTD-A), University of Milan.

Research experiences

2013.09.01 – 2017.02.28 Postdoctoral Researcher at the University of Milan — “*Construction of Invariant Manifolds via Normal Forms: from Celestial Mechanics to Hamiltonian PDE*”. Supervisor: Prof. A. Giorgilli.

2011.10.01 – 2013.08.31 FSR Postdoctoral Researcher at the University of Namur (“FSR Incoming Post-doctoral Fellowship of the Académie universitaire Louvain, cofunded by the Marie Curie Actions of the European Commission”) — “*Dynamics near invariant manifolds (DyNeInMa)*”. Supervisor: Prof. A. Lemaître.

2010.11.01 – 2011.04.30 Postdoctoral Researcher at the University of Rome “Tor Vergata” — “*Stabilità dei sistemi planetari, aspetti teorici e computazionali*”. Supervisor: Prof. U. Locatelli.

2007.11.05 – 2011.02.11 Ph.D. in Mathematics at the University of Milan with full marks and honors. Title of the thesis: “*Effective Stability of Hamiltonian Planetary Systems*”. Supervisors: Prof. A. Giorgilli and Prof. U. Locatelli.

Awards and Habilitation

2018 Italian National Scientific Habilitation as Associate Professor in “01/A4 - Fisica Matematica” (unanimous vote) — from 2018.07.03 to 2024.07.13.

2011 Awarded of a “INDAM-COFUND Fellowships in Mathematics and/or Applications for Experienced Researchers cofunded by Marie Curie” (outgoing type).

(not accepted because already beneficiary of a “FSR Incoming Post-doctoral Fellowship of the Académie universitaire Louvain, cofunded by the Marie Curie Actions”)

Research Projects

2019 Associate Investigator of the Milan Research Unit for the PRIN research project “New frontiers of Celestial Mechanics: theory and applications” (PI: Prof. Celletti).

2018 – now PI of the Progetto Giovani 2018 — INdAM-GNFM research project: “Resonant Normal Forms in Hamiltonian Systems”.

2017 – 2018 Member of the Progetto Giovani 2017 — INdAM-GNFM research project: “Normal form techniques in Lattice Dynamics and Celestial Mechanics: perturbed dynamics via resonant normal forms” (PI: Dr. Penati).

2013 – 2016 Member of the PRIN research project “Teorie geometriche e analitiche dei sistemi Hamiltoniani in dimensioni finite e infinite” (PI: Prof. Dubrovin).

2014 – now Member of the INdAM-GNFM research group.

2009 – 2011 Member of the INdAM-GNFM research group.

Organizing Experience

2014 Local Organizing Committee for the “International Astronomical Union (IAU) Symposium 310: Complex planetary systems”, 7–11 July 2014, Namur, Belgium.

2013 – now Member of the “Commissione Informatica”, Dipartimento di Matematica, Università degli Studi di Milano.

2008 – 2011 Member of the “Commissione Informatica”, Dipartimento di Matematica, Università degli Studi di Milano.

Education

2005.07.22 – 2007.07.16 M.Sc. Mathematics at the University of Milano-Bicocca with full marks and honors (110/110 cum laude). Title of the thesis: “*Stabilità nel senso di Nekhoroshev di tori KAM*”. Supervisors: Prof. D. Noja, Prof. A. Giorgilli and Prof. U. Locatelli.

2002.09.17 – 2005.07.18 B.Sc. in Mathematics at the University of Milano-Bicocca with full marks and honors (110/110 cum laude). Title of the thesis: “*Funzioni a variazione limitata*”. Supervisor: Prof. A. Cellina.

Lectures at Schools

2019 “*KAM theory in Celestial Mechanics*”, Master *Mathematical and physical methods for space science*, University of Turin, Turin, Italy.

2016 “*Programmazione su schede grafiche (GPU) in CUDA*”, Infrastrutture di Calcolo a Basso Costo (INCA-ABACO), Università di Roma “Tor Vergata”, Roma, Italia.

2011 “*Methods of algebraic manipulation in perturbation theory*”, LAPIS 2011: Third La Plata International School on Astronomy and Geophysics, La Plata, Argentina.

Graduate Teaching

2017 – now Teaching assistant of the course: “*Sistemi Hamiltoniani 1*”, charged by Prof. A. Giorgilli, Dept. of Mathematics, University of Milan.

2015 – 2016 Teaching assistant of the course: “*Laboratorio di Modellistica Matematica*”, charged by Dr. F. Ieva, Dept. of Mathematics, University of Milan.

2014 – 2016 Teaching assistant of the course: “*Laboratorio di programmazione in CUDA*”, charged by Prof. A. Giorgilli, Dept. of Mathematics, University of Milan.

2013 – 2016 Teaching assistant of the course: “*Laboratorio di Modellistica Matematica*”, charged by Prof. G. Aletti, Dept. of Mathematics, University of Milan.

2012/13 Professor in charge of the course: “*Applications des systèmes dynamique*”, Master in Mathematics, University of Namur.

Undergraduate Teaching

2018 – now Teaching assistant of the course: “*Metodi e Modelli Matematici per le Applicazioni*”, charged by Prof. S. Paleari, Dept. of Mathematics, University of Milan.

2013 – 2018 Teaching assistant of the course: “*Fisica Matematica 1*”, charged by Prof. A. Giorgilli, Dept. of Mathematics, University of Milan.

2009/10 Teaching assistant of the course: “*Metodi e Modelli Matematici per le Applicazioni*”, charged by Prof. S. Paleari, Dept. of Mathematics, University of Milan.

2008 – 2010 Teaching assistant of the course: “*Progetto MiniMat*”, Facoltà di Scienze e Tecnologie, University of Milan.

Theses supervision

Currently member of the steering committee of the Ph.D. thesis of Mara Volpi, *Analysis of the long-term stability of multi-planetary extrasolar systems and implications on their orbital characteristics*, University of Namur. Supervisor: Prof. A.-S. Libert.

Co-advisor of 7 Master Thesis in Mathematics at the University of Milan:

- V. Danesi: “Continuazione di orbite periodiche su tori risonanti” (2017).
- M. Nicoletti: “Ricerca di Orbite Periodiche nel Problema di Sitnikov” (2016).
- G. Pichierri: “Expansions in elliptic functions for highly eccentric planetary orbits” (2015).
- P. Corazza: “Evoluzione di sistemi extrasolari in risonanza” (2015).
- S. Boiani: “Stabilità dei sistemi extrasolari: analisi della dipendenza dai parametri orbitali” (2015).
- G.F. Pontoni: “Costruzione di funzioni invarianti per mappe simplettiche” (2014).
- L. Grassi: “Classical and Relativistic dynamics of extrasolar planetary systems” (2013).

Referee experiences

Referee for the journals:

- Astrophysics and Space Science;
- Celestial Mechanics and Dynamical Astronomy;
- Discrete & Continuous Dynamical Systems — Series A;
- International Journal of Bifurcation and Chaos;
- Journal of Differential Equations;
- Journal of Nonlinear Mathematical Physics;
- Mathematics in Computer Science;
- Physical Review E;
- *Reviewer for Mathematical Reviews.*

Registered in the Register of Expert Peer Reviewers for Italian Scientific Evaluation (REPRISE) Section Ricerca di Base.

References

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Papers

- [1] M. Sansottera: “*Analysis of the frequency space for particle accelerator maps: FMA and normal forms*”, (to be submitted to *Physica D*) (2019).
- [2] M. Sansottera, T. Penati, S. Paleari, V. Danesi: “*On the continuation of degenerate periodic orbits via normal form: lower dimensional resonant tori*”, (to be submitted to *CNSNS*) (2019).
- [3] T. Penati, V. Koukouloyannis, M. Sansottera, S. Paleari, P. Kevrekidis: “*On the nonexistence of degenerate phase-shift Multibreathers in a zigzag Klein-Gordon model via dNLS approximation*”, *Physica-D (revised version under review)* (2019).
- [4] D. Bambusi, A. Fusè, M. Sansottera: “*Exponential stability in the perturbed central motion*”, *RCD*, **23** 821–841 (2018).
DOI:10.1134/S156035471807002X arXiv:1705.00576
- [5] M. Volpi, U. Locatelli, M. Sansottera: “*A reverse KAM method to estimate unknown mutual inclinations in exoplanetary systems*”, *CeMDA*, **130**:36 (2018).
DOI:10.1007/s10569-018-9829-5 arXiv:1712.07390
- [6] T. Penati, M. Sansottera, V. Danesi: “*On the continuation of degenerate periodic orbits via normal form: full dimensional resonant tori*”, *CNSNS*, **61**, 198–224 (2018).
DOI:10.1016/j.cnsns.2018.02.003 arXiv:1709.07824
- [7] T. Penati, M. Sansottera, S. Paleari, V. Koukouloyannis, P. Kevrekidis: “*On the nonexistence of degenerate phase-shift discrete solitons in a dNLS nonlocal lattice*”, *Physica-D*, **370**, 1–13 (2018).
DOI:10.1016/j.physd.2017.12.012 arXiv:1707.01679
- [8] A. Giorgilli, U. Locatelli, M. Sansottera: “*Secular dynamics of a planar model of the Sun-Jupiter-Saturn-Uranus system; effective stability into the light of Kolmogorov and Nekhoroshev theories*”, *RCD*, **22**, 54–77 (2017).
DOI:10.1134/S156035471701004X arXiv:1702.04894
- [9] M. Sansottera, M. Ceccaroni: “*Rigorous estimates for the relegation algorithm*”, *CeMDA*, **127**, 1–18 (2017).
DOI:10.1007/s10569-016-9711-2 arXiv:1709.07830
- [10] M. Sansottera, A. Giorgilli, T. Carletti: “*High-order control for symplectic maps*”, *Physica-D*, **316**, 1–15 (2016).
DOI:10.1016/j.physd.2015.10.012 arXiv:1510.06561
- [11] M. Sansottera, C. Lhotka, A. Lemaître: “*Effective resonant stability of Mercury*”, *MNRAS*, **452**, 4145–4152 (2015).
DOI:10.1093/mnras/stv1429 arXiv:1510.06543
- [12] M. Sansottera, L. Grassi, A. Giorgilli: “*On the relativistic Lagrange-Laplace secular dynamics for extrasolar systems*”, *Proc. IAU Symposium S310*, 74–77 (2015).
DOI:10.1017/S174392131400787X arXiv:1510.06523
- [13] A. Giorgilli, U. Locatelli, M. Sansottera: “*Improved convergence estimates for the Schroder-Siegel problem*”, *Ann. di Mat. Pura ed Appl.*, **194**, 995–1013 (2015).
DOI:10.1007/s10231-014-0408-4 arXiv:1712.08927

- [14] A. Giorgilli, U. Locatelli, M. Sansottera: “*On the convergence of an algorithm constructing the normal form for lower dimensional elliptic tori in planetary systems*”, CeMDA, **119**, 397–424 (2014).
[DOI:10.1007/s10569-014-9562-7](#) [arXiv:1401.6529](#)
- [15] M. Sansottera, C. Lhotka, A. Lemaître: “*Effective stability around the Cassini state in the spin-orbit problem*”, CeMDA, **119**, 75–89 (2014).
[DOI:10.1007/s10569-014-9547-6](#) [arXiv:1510.06521](#)
- [16] A.-S. Libert, M. Sansottera: “*On the extension of the Laplace-Lagrange secular theory to order two in the masses for extrasolar systems*”, CeMDA, **117**, 149–168 (2013).
[DOI:10.1007/s10569-013-9501-z](#) [arXiv:1306.5624](#)
- [17] M. Sansottera, U. Locatelli, A. Giorgilli: “*On the stability of the secular evolution of the planar Sun-Jupiter-Saturn-Uranus system*”, Math. and Comp. in Sim., **88**, 1–14 (2013).
[DOI:10.1016/j.matcom.2010.11.018](#) [arXiv:1010.2609](#)
- [18] A. Giorgilli, M. Sansottera: “*Methods of algebraic manipulation in perturbation theory*”, Asociacion Argentina de Astronomia, **3**, 147–183 (2011).
<http://adsabs.harvard.edu/abs/2011WSAAA...3..147G> [arXiv:1303.7398](#)
- [19] M. Sansottera, U. Locatelli, A. Giorgilli: “*A Semi-Analytic Algorithm for Constructing Lower Dimensional Elliptic Tori in Planetary Systems*”, CeMDA, **111**, 337–361 (2011).
[DOI:10.1007/s10569-011-9375-x](#) [arXiv:1010.2617](#)
- [20] A. Giorgilli, U. Locatelli, M. Sansottera: “*Su un’estensione della teoria di Lagrange per i moti secolari*”, Rend. Ist. Lom., **143**, 221–238 (2010).
[arXiv:1303.7392](#)
- [21] A. Giorgilli, U. Locatelli, M. Sansottera: “*Kolmogorov and Nekhoroshev theory for the problem of three bodies*”, CeMDA, **104**, 159–173 (2009).
[DOI:10.1007/s10569-009-9192-7](#) [arXiv:1303.7395](#)
- [22] M. Sansottera: “*Effective Stability of Hamiltonian Planetary Systems*”, Ph.D. Thesis (supervisors: A. Giorgilli and U. Locatelli), Università degli Studi di Milano (2011).
[DOI:10.13130/sansottera-marco_phd2011-02-11](#)

Talks & posters

- [t1] “*A reverse KAM method to estimate unknown mutual inclinations in exoplanetary systems*”, Prospectives in Hamiltonian dynamics, Venice, Italy (2018).
- [t2] “*A reverse KAM method to estimate unknown mutual inclinations in exoplanetary systems*”, Assemblea Scientifica G.N.F.M., Montecatini Terme, Pistoia, Italy (2018).
- [t3] “*On the continuation of degenerate periodic orbits via normal form: full dimensional resonant tori*”, naXys seminar, Namur, Belgium (2017).
- [t4] “*Analytical treatment of long-term evolution of extrasolar systems: an extension of the classical Laplace-Lagrange secular theory*”, CELMEC VII, San Martino al Cimino, Viterbo, Italy (2017) [**keynote speaker**].

- [t5] “*Quasi-convexity of the Hamiltonian for non Harmonic or non Keplerian central potentials*”, naXys seminar, Namur, Belgium (2017).
- [t6] “*High-order control for symplectic maps*”, Computational perturbative methods for Hamiltonian systems — Applications in physics and astronomy, Athens, Greece (2016) **[invited talk]**.
- [t7] “*Rigorous Results on the Relegation Algorithm and Applications via Algebraic Manipulation*”, AstroNet-II International Final Conference, Tossa de Mar, Spain (2015) **[invited talk]**.
- [t8] “*Secular dynamics of extrasolar-systems*”, Complex Planetary Systems (IAU Symposium), Namur, Belgium (2014).
- [t9] “*Improved convergence estimates for the Schröder-Siegel problem*”, Assemblée Scientifica G.N.F.M., Montecatini Terme, Pistoia, Italia (2014).
- [t10] “*Lower dimensional elliptic tori in planetary systems via normal form*”, CELMEC VI, San Martino al Cimino, Viterbo, Italia (2013).
- [t11] “*Effective stability around the Cassini state in the spin-orbit problem*”, CELMEC VI, San Martino al Cimino, Viterbo, Italia (2013) [e-poster].
- [t12] “*Non-linear oscillations and long-term evolution, application to planetary systems and spin-orbit problem*”, Planetary Motions, Satellite Dynamics, and Space-ship Orbits, CRM Montreal, Canada (2013) **[invited talk]**.
- [t13] “*Secular Evolution of Extrasolar Planetary Systems: an Extension of the Laplace-Lagrange Secular Theory*”, American Astronomical Society Division on Dynamical Astronomy (DDA 2013), Paraty, Brazil (2013).
- [t14] “*On the secular evolution of extrasolar planetary systems*”, Tenth Workshop on Interactions Between Dynamical Systems and Partial Differential Equations (JISD2012), Barcelona, Spain (2012).
- [t15] “*On the secular evolution of extrasolar planetary systems*”, Annual Meeting Graduate School Complex, Bruxelles, Belgium (2012).
- [t16] “*Explicit Construction of Elliptic Tori for Planetary Systems*”, 8th Alexander von Humboldt Colloquium for Celestial Mechanics, Bad Hofgastein, Salzburg, Austria (2011).
- [t17] “*Effective Stability of Hamiltonian Planetary Systems*”, Sistemi dinamici nonlineari e applicazioni, Pisa, Italy, (2011).
- [t18] “*Explicit Construction of Elliptic Tori for Planetary Systems*”, Applications of Computer Algebra (ACA’10), Vlora, Albania (2010).
- [t19] “*Explicit construction of elliptic tori for planetary systems*”, Emerging Topics in Dynamical Systems and Partial Differential Equations, Barcelona, Spain (2010) [poster].
- [t20] “*Towards stability results for planetary problems with more than three bodies*”, Computer Algebra and Differential Equations (CADE 2009), Pamplona, Spain (2009) **[invited talk]**.
- [t21] “*Risultati sulla stabilità per problemi planetari con più di tre corpi*”, Assemblée Scientifica G.N.F.M, Montecatini Terme, Pistoia, Italy (2009).
- [t22] “*Towards stability results for planetary problems with more than three bodies*”, CELMEC V, San Martino al Cimino, Viterbo, Italia (2009).

Research Activity

The Problems and the objectives

Werner Heisenberg once said: “...*the progress of physics will to a large extent depend on the progress of nonlinear mathematics and of methods to solve nonlinear equations...*”.

Nonlinear equations are at the basis of fundamental physical phenomena, among others:

- Celestial Mechanics (n -body and spin-orbit problem);
- Hamiltonian lattices dynamical systems (chains of weakly coupled oscillators like the Klein-Gordon (KG) and the discrete nonlinear Schrödinger (dNLS) equation);
- *a-priori* control of symplectic maps (e.g., particle accelerator maps).

All these problems fall into the so-called general problem of dynamics: a nearly-integrable Hamiltonian system $H(p, q) = h_0(p) + \varepsilon h_1(p, q)$ with action-angle variables (p, q) and ε small parameter. For $\varepsilon = 0$ the problem is *integrable* and the solution is trivial: the actions of the system are conserved quantities and the motion is periodic or quasi-periodic. Instead, in general, for $\varepsilon \neq 0$ the problem cannot be solved in closed form and the system exhibits the coexistence of regular and chaotic behaviors.

Nonlinear equations are extremely difficult to solve and perturbation techniques proved to be very effective. The major achievements of modern perturbation theory might be seen as a generalization of periodic orbits that, as said by Poincaré, are essentially the only way in which we can try to enter a place that before was considered inaccessible. In particular the two main milestones are:

- the Kolmogorov-Arnold-Moser (KAM) theorem on the persistence of invariant tori in nearly integrable Hamiltonian system;
- the Nekhoroshev theorem on the bounds of the actions over exponentially long times, namely the exponential bounds of the so-called Arnold’s diffusion.

Mathematicians and Physicists have made lot of progress in these subjects in the last decades, especially on the actual applicability of the KAM and Nekhoroshev theories to *realistic* models, e.g., the giant planets of our Solar system. However, there is still a big gap between numerical investigations and the theory that needs to be filled. Indeed, the purely analytic results are not enough in order to get realistic estimates. The first attempt to apply the KAM theorem to prove the stability of the Solar System was performed by Hénon in 1965, who found that the mass ratio between Jupiter and Sun should be smaller than 10^{-320} . Quoting Hénon: “Thus the theorem has only a theoretical interest and is absolutely not of practical use, at least in the presented form”.

The key remark is that a practical application of the theory to a realistic system needs an explicit constructive algorithm that can be effectively implemented using computer algebra. On the other hand, as suggested by Poincaré, the constructive method should be based on a rigorous mathematical framework. This kind of approach (also implementing interval arithmetic) opens a complete new field in Celestial Mechanics and allowed some authors (among the others Calleja, Celletti, Chierchia, de la Llave, Gabern, Giorgilli, Jorba, Locatelli, Simó) to (rigorously) prove the existence of KAM tori for some interesting problems in Celestial Mechanics.

In my research activity I address problems arising in different fields of Hamiltonian Mechanics using a common constructive perturbative approach based on normal form theory. The key ingredients are:

- the construction of suitable invariant (or approximately invariant) manifolds;
- the use of computer algebra in order to explicitly perform the computations;
- the *effective stability* in the sense of Nekhoroshev.

The strongest and most difficult point of my research project is the attempt to obtain *realistic results* implementing rigorous mathematical methods. In order to make clear the differences between a theoretical and an applied result, let me stress that *in a physical problem the value of the small parameter ε is given by Nature* and has a fixed value, e.g., in the planetary problems it essentially represents the mass ratio between the star and the biggest planet. I deal with both the theoretical and computational aspects, with a special care on their interplay. The applications to realistic models are obtained by translating the *explicit* normal form algorithms into symbolic computations that are implemented via a specifically designed algebraic manipulator (see [18] for an introduction to the main ideas that have been translated in our codes).

My research activity mainly concerns the following topics:

- (a) Celestial Mechanics;
- (c) Hamiltonian lattices dynamical systems;
- (d) Maps in a neighborhood of an equilibrium.

The research results have been published in international publications and presented in international conferences. For the complete lists of the papers and talks, please refer to the subsequent sections “Papers” and “Talks & posters”, respectively.

Original contributions

(a.1) Celestial Mechanics — secular dynamics of the giant planets of the Solar system

A productive combination of KAM and Nekhoroshev theories consists in applying the usual, local theory for an elliptic equilibrium to the neighbourhood of an invariant Kolmogorov torus. This is exactly the problem I tackled in my Ph.D. thesis^[22]. The results of my Ph.D. thesis have been collected in [21], [20],[19] and [17], thus [22] has not been included in the list of the 12 selected papers (i.e., Allegato C).

In [21] we investigated the long-time stability for the Sun-Jupiter-Saturn (SJS) system in the framework of the three-body problem. We started from a previous result on the existence of a torus for the SJS system (see Locatelli and Giorgilli, DCDS-B, 7, 2007) based on the explicit expansion of the Hamiltonian and on the explicit application of Kolmogorov method up to a finite, not too low order. Then we worked out a Birkhoff normalization and showed that there is a domain of effective stability, which is centered around an invariant KAM torus. The results were close to realistic ones.

In [17] we studied the stability of the secular evolution of the planar Sun-Jupiter-Saturn-Uranus (SJSU) system. Our method may be considered as a major refinement of the Lagrange theory for the secular motions. Indeed, we improved the classical circular approximation by replacing it with a torus which is invariant up to order two in the masses. Therefore, we investigated the stability of the elliptic equilibrium point of the secular system for small values of the eccentricities. For the initial data corresponding

to a real set of astronomical observations, we found an estimated stability time of 10^7 years, which is not extremely far from the estimated lifetime of the Solar System.

A similar approach was applied to the planar SJS system in [20], where a technical improvement concerning the bound of a polynomial function in a poly-disk allowed us to get a better estimated stability time.

The results obtained in the above-quoted works show that in order to better study the long-time stability of a planetary system, one should find an approximated invariant object that is as close as possible to the dynamics described by the system. Concerning the planetary orbital revolutions, the classical approach consists in taking the circular orbit as a reference. However, due to the effects of near-resonances between the planets (e.g., Jupiter and Saturn are close to the 5:2 mean-motion resonance, the so-called *great inequality*) one should replace the circular approximation with a torus that is invariant up to order two in the masses, using a Kolmogorov-like procedure. This allows to study the stability of the secular system for rather small values of the eccentricities. Coming to the secular evolution, the simplest approach consists in the study of the dynamics around the elliptic equilibrium (i.e., the Lagrange-Laplace secular theory). A refined approach consists in replacing the elliptic equilibrium with a KAM torus, which approximates very well the secular orbits.

The natural extension consists in looking for an invariant object that replaces the circular approximation and the invariant torus at order two in the masses, namely an elliptic lower dimensional invariant torus. The existence of elliptic lower dimensional invariant tori is not a straightforward consequence of Kolmogorov's theorem and all available theorems (see, e.g., Pöschel, Math. Z., 202, 1989) are not suitable for explicit calculations, even if one is interested just in finding the locations of the elliptic invariant tori, being clever adaptations of Arnold's proof of KAM theorem.

In [19] we devised an original semi-analytic algorithm for the construction of lower dimensional elliptic tori in planetary systems, following the original Kolmogorov scheme. Moreover we applied our algorithm in order to construct an elliptic torus for a planar model of the SJSU system. Finally, by using the frequency analysis method, we verified that our location of the initial conditions on an invariant elliptic torus was really accurate. This semi-analytic algorithm has been supported with rigorous convergence estimates in [14], where we gave a constructive proof of the existence of elliptic lower dimensional tori in nearly-integrable Hamiltonian systems. In particular we adapted the classical Kolmogorov normalization algorithm to the case of planetary systems. With respect to previous works on the same subject we exploited the characteristic of Lie series giving a precise control of all terms generated by our algorithm. This allowed us to slightly relax the non-resonance conditions on the frequencies.

In [8] we investigated again the long-time stability of a planar model for the SJSU system. In particular we improved the results in [17] by using a similar approach to the one adopted in [21]. First, we explicitly constructed a Kolmogorov normal form, so as to find an invariant KAM torus accurately which approximates the secular orbits. Then, we adapted the approach at the basis of the analytic part of the Nekhoroshev's theorem, so as to show that there is a neighborhood of that torus for which the estimated stability time is larger than the lifetime of the Solar System. The size of such a neighborhood,

compared with the uncertainties of the astronomical observations, is not far from the real physical parameters.

(a.2) Celestial Mechanics — spin-orbit problem, Titan and Mercury

Like the Moon, most of the regular satellites of the Solar System present the same face to their planet. Cassini (1693), considering a simplified model of the rotation of the Moon, showed how this peculiar feature corresponds to an equilibrium, called a Cassini state. Moreover, he investigated the effects of small perturbations, namely the possible destabilization of this equilibrium or just to the excitation of librations around it.

In [15] we investigated the long-time stability in the neighborhood of the Cassini state in the conservative spin-orbit problem. We constructed a high-order Birkhoff normal form and gave an estimate of the effective stability time in the Nekhoroshev sense. By extensively using algebraic manipulations on a computer, we explicitly applied our method to the rotation of Titan, the largest moon of Saturn, that is in 1:1 spin-orbit resonance. We obtained physical bounds of Titan's latitudinal and longitudinal librations, finding a stability time greatly exceeding the estimated age of the Universe. In addition, we studied the dependence of the effective stability time on three relevant physical parameters: the orbital inclination, i , the mean precession of the ascending node of Titan orbit, $\dot{\Omega}$, and the polar moment of inertia, C .

In [11] we extended our investigation to Mercury, the unique known planet that is currently situated in a 3:2 spin-orbit resonance. Specifically, we used the same approach adopted for the 1:1 spin-orbit case, with a peculiar attention to the role of Mercury's non negligible eccentricity.

(a.3) Celestial Mechanics — extrasolar planetary systems

The first confirmation of an exoplanet was made in 1995 and nowadays more than 100 multi-planetary systems have been discovered. The discovery of extrasolar planetary systems has opened a new field in Celestial Mechanics. The study of extrasolar system raised two particularly relevant problems: (i) most exoplanets have highly eccentric orbits, in contrast with the almost circular orbits of the Solar System; (ii) there are many giant planets orbiting at a low distance from the central star, with periods of a few months or even a few days. In the latter case relativistic effects could have a significant impact and should be taken into account.

In [16] we studied the secular evolution of several exoplanetary systems by using a major refinement of the classical Laplace-Lagrange, i.e., an approximation at order two in the masses. The aim of the work was to reconstruct the evolution of the eccentricities and pericenters of the planets by using analytic techniques. Our study clearly showed that, for systems close to a mean-motion resonance, the second order approximation described their secular evolution much more accurately than the usually adopted first order one. Moreover, this approach took into account the influence of the mean anomalies on the secular dynamics. Furthermore, as a byproduct of the approximation at order two in the masses, we also gave an estimate of the proximity to a mean-motion resonance of the two-planet extrasolar systems discovered so far. In particular we introduce a simple analytic criterion that allowed to discriminate between three different categories of planetary systems: secular, *near* mean-motion resonance and *in* mean-motion resonance.

In [12] we extended the Lagrange-Laplace secular theory to high order and also include the main relativistic effects. Specifically, we investigated the long-term evolution of the planetary eccentricities via normal form and we found an excellent agreement with direct numerical integrations. Finally we set up a simple analytic criterion that allows to evaluate the impact of the relativistic effects in the long-term evolution.

Another open problem concerns the inclinations of exoplanets. Indeed, for systems detected via radial velocity method, the inclinations are essentially unknown. In [5] we provided estimations of the ranges of mutual inclinations that were compatible with the long-term stability of the system. Focusing on the skeleton of an extrasolar system, i.e., considering only the two most massive planets, we studied the Hamiltonian of the three-body problem after the reduction of the angular momentum. Such a Hamiltonian was expanded both in Poincaré canonical variables and in the small parameter D_2 , which represents the normalized Angular Momentum Deficit. The value of the mutual inclination was deduced from D_2 and, thanks to the use of interval arithmetic, we were able to consider open sets of initial conditions instead of single values. Looking at the convergence radius of the Kolmogorov normal form, we developed a *reverse KAM approach* in order to estimate the ranges of mutual inclinations that were compatible with the long-term stability in a KAM sense. Our method was successfully applied to the extrasolar systems HD 141399, HD 143761 and HD 40307.

(b) Hamiltonian lattices dynamical systems

Many physical system, e.g., vibrations of molecular crystals and biomolecules including DNA-chains, are well modeled as Hamiltonian network of weakly coupled anharmonic oscillators. Localization is nowadays a well-known phenomenon in nonlinear lattices and it is also known that for small enough coupling or large enough amplitude the system admits families of periodic solutions exponentially localized in space, the so-called breathers. The breathers were experimentally observed and explored in many physical systems, including, among others, nonlinear magnetic meta-materials, electrical lattices, Bose-Einstein condensates and chains of mechanical oscillators. The generalization of the breather solution is the so-called multibreather, solutions that are localized on more than one site of the lattice. The existing knowledge on properties and especially on the stability of the multibreathers is limited to the small coupling limit, the so-called anticontinuous limit, and to nondegenerate solutions.

In [7] we studied the problem of the continuation of degenerate multibreathers solutions, obtaining the nonexistence of any asymmetric vortex solution in the one-dimensional discrete nonlinear Schrödinger lattices. First, we exploited the presence of a conserved quantity for the soliton profile (the so-called density current), together with a perturbative construction, and proved the nonexistence of any asymmetric vortex solution which is at least C^2 with respect to the small coupling ε . Then, under less restrictive assumptions, nonexistence was proved by studying the bifurcation equation of a Lyapunov-Schmidt reduction, expanded to suitably high orders.

In [3] we studied the existence of low amplitude four-site phase-shift multibreathers for small values of the coupling in Klein-Gordon chains with interactions longer than the classical nearest-neighbour ones. We focused mainly in the case of interactions up to next-to-nearest neighbours, which, in the proper parameter regime, is

equivalent to the zigzag configuration. Initially we examined the persistence conditions of the system, in order to seek for vortex-like motions. Although this approach provided a useful insight, due to the degeneracy of these solutions, it did not allow us to determine if they consisted true solutions of our system. In order to overcome this obstacle, we proceeded to a deeper mathematical analysis. By means of a Lyapunov-Schmidt decomposition we were able to establish that the bifurcation equation for our model could be considered, in the small energy and small coupling regime, as a perturbation of the one of a corresponding non-local discrete nonlinear Schrödinger equation.

In [6] we reconsidered the classical problem of the continuation of degenerate periodic orbits in Hamiltonian systems. In particular we focused on periodic orbits that arose from the breaking of a completely resonant maximal torus. We proposed a suitable normal form construction that allowed to identify and approximate the periodic orbits which survived to the breaking of the resonant torus. Our algorithm allowed us to treat the continuation of approximate orbits which were at leading order degenerate, hence not covered by classical averaging methods.

(c.1) Maps in a neighborhood of an equilibrium — the Schröder-Siegel problem

In [13] we reconsidered the Schröder-Siegel problem of conjugating an analytic map in \mathbb{C} in the neighborhood of a fixed point to its linear part, extending it to the case of dimension $n > 1$. Assuming a condition which is equivalent to Bruno's one on the eigenvalues $\lambda_1, \dots, \lambda_n$ of the linear part we showed that the convergence radius ϱ of the conjugating transformation satisfied $\ln \varrho(\lambda) \geq -C\Gamma(\lambda) + C'$ with $\Gamma(\lambda)$ characterizing the eigenvalues λ , a constant C' not depending on λ and $C = 1$. This improved the previous results for $n > 1$, where the known proofs give $C = 2$ (recall that $C = 1$ is known to be the optimal value for $n = 1$).

(c.2) Maps in a neighborhood of an equilibrium — high-order (a priori) control

In [10] we revisited the problem of introducing an *a priori control* for devices that can be modeled via a symplectic map in a neighborhood of an elliptic equilibrium, e.g., a particle accelerator. Given a symplectic map, the problem was to add a (small) non-trivial control term such that the resulting modified map is conjugated to a rotation, possibly a twist one. Using a technique based on Lie transform methods we produced a normal form algorithm that avoided the usual step of interpolating the map with a flow. The formal algorithm was completed with quantitative estimates that brought into evidence the asymptotic character of the normal form transformation. In addition, we discussed how control terms of different orders might be introduced so as to increase the size of the stable domain of the map.

The relegation algorithm

In [9] we revisited the relegation algorithm introduced by Deprit et al. (CeMDA, 79, 2001). This relatively recent algorithm is nowadays widely used for implementing closed form analytic perturbation theories, as it generalizes the classical Birkhoff normalization algorithm. Following the usual tradition in Celestial Mechanics, the relegation algorithm was introduced and used in a *formal* way, i.e. without providing any rigorous convergence or asymptotic estimates. In this work we supported the formal algorithm with rigorous quantitative estimates and showed how the results about stability over exponentially

long times could be recovered in a simple and effective way, at least in the non-resonant case.

Perturbed Central Motion

In [4] we revisited the spatial central problem with a real analytic potential, proving that the corresponding Hamiltonian, when written in action-angle variables, was almost everywhere quasiconvex, the only exceptions being the Keplerian and the Harmonic potentials. We deduced a Nekhoroshev type stability result for the perturbed spatial central motion. The case where the central system was put in interaction with a slow system was also studied and stability over exponentially long time was proved.

Future plans

(1) Stability of some subsystems of our Solar System

(1.1) Restricted quasi-periodic problem

Several simplifications of the 3-body (and 4-body) problem have been considered in the literature, e.g., the circular or elliptic restricted three-body problem, the bicircular and quasi-bicircular models. We plan to introduce a *restricted quasi-periodic* problem in a non-inertial frame, based on previous work on the problem of three bodies. The idea is to exploit the fact that Sun, Jupiter and Saturn exhibits quasi-periodic motions on invariant KAM tori very close to the real initial conditions. This will make the Hamiltonian model much more tractable.

(1.2) Stability of the Sun-Jupiter-Saturn-Uranus-Neptune system

A fruitful combination of KAM and Nekhoroshev theory has been exploited for the study of the *effective stability* of the Sun-Jupiter-Saturn and Sun-Jupiter-Saturn-Uranus systems, see [21], [17] and [8]. These are the best available current rigorous results. Still we plan to push beyond these achievements both concerning the estimated effective stability time and the inclusion of Neptune into the model.

(1.3) Long-time stability around lower dimensional elliptic tori

The starting point is the explicit constructed lower dimensional elliptic torus for the planar SJSU system, see [19] and [14]. Replacing the classical circular approximation with this invariant manifold is definitely a better starting point for a perturbative approach. We plan to investigate the long-time stability of the SJSU system in a neighborhood of the lower dimensional elliptic torus.

(2) Exoplanets Dynamics

(2.1) Accurate long-term behavior of exoplanets: 3D and MMR

Recently, jointly with Libert, we produced an accurate description of the long-term dynamics of coplanar exoplanets that are in a 2:1 mean-motion resonance (MMR), extending our previous work [16]. We plan to extend that achievement to any kind of MMR, to the spatial case and to system with more than two planets.

(2.2) Determination of unknown orbital parameters via inverse KAM

Stability is a natural requirement for an analytic model consistent with observations. Indeed, there is a very low probability of detect unstable extrasolar planetary systems. We aim at exploiting this idea: assuming that the system is stable for a long

time, e.g., the orbits lie on a KAM torus, confine the *unknown* orbital parameters, e.g., the mutual inclination between two planets. Specifically, we intend to link possible values of the orbital parameters to the radius of convergence of the Kolmogorov normal form. This idea was recently developed in [5], but further refinements are needed in order to face systems with high eccentricity.

(2.3) Expansions of elliptic motion via elliptic function theory

In collaboration with Pichierri and Giorgilli, we obtained some preliminary results concerning the expansions of elliptic motion via elliptic functions. It turns out that this kind of expansion are convergent for any value of the eccentricity, and not narrowed by the Laplace limit ($e < 0.6627\dots$), thus they are in a much better position with respect to the classical expansions for applications to extrasolar systems. We plan to replace the classical expansions with these new ones based on elliptic functions. The drawbacks are twofold: first one has to leave the Hamiltonian framework and directly works on the differential equations, second one has to deal with non-elementary elliptic functions in place of the usual polynomials and trigonometric polynomials. This is the price to pay, but the benefits are far superior.

(2.4) Effective stability of extrasolar planetary systems

The stability analysis of extrasolar planetary systems represents a key ingredient for understanding the wider question of the architecture of planetary systems. The experience acquired studying the stability of our Solar system will guide the development of this line of research.

(3) Hamiltonian lattices dynamical systems

(3.1) Lower dimensional tori in weakly coupled anharmonic oscillators

Recently in [6] we investigated the continuation of degenerate periodic orbits on a completely resonant torus with respect to a small parameter, introducing an original normal form algorithm. The normal form algorithm that we have developed represents an excellent starting point for the extension to completely resonant lower dimensional tori. This will allow to deal with degenerate scenarios which emerge studying discrete solitons in one-dimensional nonlocal discrete nonlinear Schrödinger lattices and in the investigation of vortexes in two-dimensional square lattices

(3.2) Nonlinear stability of breathers and multibreathers

The problem of stability of breathers has attracted a lot of interest since their discovery and many studies has been devoted to their nonlinear stability. We aim to study the effective long-time stability in a neighborhood of breathers (or multibreathers) exploiting the constructive normal form approach previously described. Furthermore, we intend to exploit the constructive nature of the normal forms algorithms and study the actual long-time stability of a *physically meaningful* system of weakly coupled anharmonic oscillators, comparing the results with the observations relative to experiments performed in many physical systems.

(3.3) Normal form at the thermodynamic limit

It is well-known that results like the KAM and the Nekhoroshev theorems stated for finite dimensional systems appear to be somewhat useless as the number N of degrees of freedom of the system grows. Indeed the estimated dependence on N of the constants

involved is usually very bad, and in particular the (specific) energy thresholds do vanish in the limit $N \rightarrow \infty$. We plan to extend the results of Giorgilli et al. (Annales Henri Poincaré, 16, 2015) concerning the existence of an extensive adiabatic invariant in the thermodynamic limit in dimension greater than one.

(4) Maps in a neighborhood of an equilibrium

(4.1) Control of symplectic maps and multiturn extraction

Symplectic maps are intensively used in order to describe the nonlinear particle motions in circular accelerators and recently a novel approach has been proposed at CERN for performing multiturn extraction from a circular machine by using stable resonant islands. In [10] we studied the the problem of introducing an *a priori* high-order control for devices that can be modeled via a symplectic map in a neighborhood of an elliptic equilibrium, like particle accelerators. This result goes much beyond the usually adopted first order corrections and, in principle, it allows a lot of flexibility depending on the type of control one is interested in. We plan to revisit that approach. First concerning the application to *realistic* maps that accurately model the behavior of particle in circular accelerators. Second, to exploit the control term in order to introduce a *wanted resonance* in such a way to maximize the dynamical aperture of the islands in connection to the multiturn extraction techniques.

(4.2) KAM theory for mappings

It is well-known that transporting the analytical methods of normal form theory from differential equations to maps is not straightforward. Indeed most of the available proofs of KAM theorem for nearly integrable analytic symplectic have been proved exploiting the so-called interpolating Hamiltonian which generates the symplectic map. A recent work by Giorgilli (Rend. Ist. Lomb., 146, 2012) solves the problem of the representation of maps by Lie transforms. This allowed us to improve the convergence estimates for the Schröder-Siegel problem extending the results by Yoccoz to any finite dimensions [13]. We plan to give a direct proof of a KAM theorem for maps, without using the classical interpolating Hamiltonian.

(4.3) Nekhoroshev theory for mappings

This part of the project make a match with the previous one. Indeed also the available Nekhoroshev proofs make use of the interpolating Hamiltonian, with the exception of the approach designed in Guzzo (Annales Henri Poincaré, 5, 2004), which is certainly much less suitable to implement explicit calculations, with respect to a scheme based on Lie series/transforms. We aim to prove a Nekhoroshev theorem for maps, without using the classical interpolating Hamiltonian.

Additional Information

Programming languages

C, CUDA-C, FORTRAN, MPI and OpenMP.

Mathematical Packages

Mathematica, Maxima, Matlab, Octave, T_EX and L^AT_EX.

Operating Systems

GNU/Linux and Windows.

Languages

Italian (mothertongue); English (fluent); French (intermediate level).

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