# Curriculum vitae et studiorum

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Allegato B

Università degli Studi di Milano
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# 1 Basic information

## 1.1 Personal details

ERTL-BLEIMHOFER VERONIKA
Born on the 28th of December 1983

## 1.2 Work place address and email

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00-656 Warszawa
Poland
Email: vertl@impan.pl

## 1.3 Languages

| | |
|---|---|
| **German** | native language |
| **French** | fluent (reading, writing, speaking) |
| **English** | fluent (reading, writing, speaking) |
| **Latin** | examen latinum (translation, reading, writing) |
| **Italian** | elementary (reading, writing, speaking) |
| **Japanese** | elementary (reading, writing, speaking) |
| **Spanish** | elementary (reading, writing, speaking) |
| **Polish** | elementary (reading, writing, speaking) |

## 1.4 Current employment

| | |
|---|---|
| Oct. 2023 – present | **IMPAN**, Warszawa, Poland<br>postdoc in the group of Prof. P. Achinger |

## 1.5 Previous employment

| | |
|---|---|
| Oct. 2020 – Sept. 2023 | **Universität Regensburg**, Regensburg, Germany<br>Akademische Rätin auf Zeit (postdoc), group of Prof. N. Naumann |
| Aug. 2018 – Sept. 2020 | **Universität Regensburg**, Regensburg, Germany<br>Akademische Rätin auf Zeit (postdoc), group of Prof. N. Naumann |
| Aug. 2017 – July 2018 | **Keio University**, Yokohama, Japan<br>KiPAS Arithmetic and Number Theory, JSPS postdo, group of Prof. K. Bannai |
| May 2016 – July 2017 | **Universität Regensburg**, Regensburg, Germany<br>Akademische Rätin auf Zeit (postdoc), group of Prof. N. Naumann |
| Mar. 2014 – April 2016 | **Universität Regensburg**, Regensburg, Germany<br>postdoc in the DFG research training group **GRK 1692** "Curvature, cycles and cohomology", group of Prof. N. Naumann |

### 1.6 Research visits

| | |
|---|---|
| December 2023 | **Unicersità degli Studi di Milano**, Italy<br>project with Prof. Alberto Vezzani |
| May 2022 – June 2022 | **Isaac Newton Institute**, Cambridge, UK<br>Program "$K$-theory, algebraic cycles and motivic homotopy theory" |
| December 2022 | **Université de Picardie Jules Verne**, Amiens, France<br>project with MCF R. Abdellatif |
| March 2022 | **Université de Picardie Jules Verne**, Amiens, France<br>project with MCF R. Abdellatif |
| Febr. 2020 – Mar. 2020 | **Isaac Newton Institute**, Cambridge, UK<br>Program "$K$-theory, algebraic cycles and motivic homotopy theory" |
| Mar. 2019 – May 2019 | **Keio University**, Yokohama, Japan<br>group of Prof. Bannai |
| Apr. 2017 – June 2017 | **Keio University**, Yokohama, Japan<br>KiPAS Arithmetic and Number Theory, group of Prof. K. Bannai |
| Apr. 2017 | **Institut Mittag-Leffler**, Stockholm, Sweden<br>Program "Algebro-Geometric and Homotopical Methods" |
| Oct. 2012 – Dec. 2012 | **École Normale Supérieure**, Lyon, France<br>group of Prof. Wiesława Nizioł |

## 2 Academic career

### 2.1 Degrees

**Universität Regensburg** Regensburg, Germany, 2016 – 2021
Habilitation in Mathematics, facultas docendi
Thesis: "Constructions and Applications in $p$-adic Cohomology Theories"
Referees: Profes. M. Gros, M. Olsson

**University of Utah** Salt Lake City, USA, 2009 – 2014
PhD in Mathematics
Dissertation: "Overvonvergent Chern classes and higher cycle classes"
Advisor: Prof. W. Nizioł
Jury: Profes. W. Nizioł, T. de Fernex, Y.-P. Lee, G. Savin, P. Roberts

**Ludwig–Maximilians Universität** Munich, Germany, 2007 – 2010
Diploma in mathematics, minor: theoretical physics
Thesis: "Fermat's Last Theorem and the Modularity Theorem"
Advisor: F. Morel

**Université Paris 13 (Nord)** Villetaneuse, France, 2006 – 2007
Master 1 in mathematics
Thesis: "Les groupes $p$-divisibles"
Advisor: O. Brinon

**Ludwig–Maximilians Universität** Munich, Germany, 2003 – 2006
Pre-diploma in mathematics, minor: theoretical physics

### 2.2 References

The following people (in alphabetic order) have written and are willing to write letters of recommendation on my behalf:

**Kenichi Bannai** Keio University, Department of Mathematics (Japan)
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**Kiran S. Kedlaya** University of California in San Diego, Department of Mathematics (USA)
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**Bernard Le Stum** Institut de Recherche Mathématiques de Rennes (France)
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**Atsushi Shiho** University of Tokyo, Department of Mathematics (Japan)
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**Sarah Zerbes** ETH Zürich, Department of Mathematics (Switzerland)
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# 3 Research merits

## 3.1 Research area

I undertake research in arithmetic geometry, in particular I research $p$-adic cohomology theories and their applications. The techniques I use frequently come from Category Theory, Combinatorics, Commutative Algebra, Homotopy Theory, Derived Algebraic Geometry, Homological Algebra, Topology, just to name a few. Arithmetic Geometry is fascinating to me because it is so versatile, provides (more than) a starting point for interdisciplinary work, and the possibility to grow in different directions.

One of my mathematical dreams is to fully understand the relation between different tools that occur in the research of special values of $p$-adic $L$-functions, more precisely syntomic and motivic cohomology.

I am also interested in the study of integral $p$-adic cohomology theories, in particular in the construction of an integral $p$-adic cohomology theory for open varieties.

*Key words*: $p$-adic cohomology theories, syntomic cohomology, motivic cohomology, descent, derived geometry.

## 3.2 Publications

### 3.2.1 Peer-reviewed publications

[1] Ertl V.: **A new proof of a vanishing result due to Berthelot, Esnault, and Rülling**.
Journal of Number Theory, vol. 237, pp. 242–256, (2022).

[2] Ertl V. et Yamada K.: **Comparison between rigid and crystalline syntomic cohomology for strictly semistable log schemes with boundary**.
Rendiconti del Seminario Matematico della Università di Padova, vol. 145, pp. 213–291, (2021).

[3] Ertl V. et Shiho A.: **On infiniteness of integral overconvergent de Rham–Witt cohomology modulo torsion**.
Tohoku Mathematical Journal, vol. 72, no. 3, pp. 395–410, (2020).

[4] Ertl V. et Miller L.E.: **Witt differentials in the $h$-topology**.
Journal of Pure and Applied Algebra, vol. 223, no. 12, pp. 5285–5309, (2019).

[5] Ertl V. et Nizioł W.K.: **Syntomic cohomology and $p$-adic motivic cohomology**.
Algebraic Geometry, vol. 6, no. 1, pp. 100–131, (2019).

[6] Ertl V. et Sprang J.: **Integral Comparison of Monsky–Washnitzer and overconvergent de Rham–Witt cohomology**.
Proceedings of the AMS, Series B, vol. 5, pp. 64–72, (2018).

[7] Ertl V.: **Full faithfulness for overconvergent $F$-de Rham–Witt connections**.
Comptes Rendus Mathématiques, vol. 354, no. 7, pp. 653–658, (2016).

#### 3.2.2 Preprints

[1] Ertl V., Shiho A. et Sprang J.: **Integral p-adic cohomology theories for open and singular varieties**.
Available at arXiv:2105.11009, (2021).

[2] Ertl V. et Yamada K.: **Poincaré duality for rigid analytic Hyodo–Kato theory.**
Available at arXiv:2009.09160, (2020).

[3] Ertl V. et Yamada K.: **Rigid analytic reconstruction of Hyodo–Kato theory**.
Available at arXiv:1907.10964, (2019).

#### 3.2.3 Other academic articles

[1] Ertl V.: **Witt differentials and the $h$-topology**.
Oberwolfach Report 6/2019, DOI: 10.4171/OWR/2019/6.

[2] Ertl V.: **Comparison theorems between overconvergent and rigid cohomology with coefficients**.
Oberwolfach Report 32/2014, 1784–1786.

#### 3.2.4 Dissertation

[1] Ertl V.: **Overconvergent Chern classes and higher cycle classes**.
Dissertation, University of Utah, College of Science, Department of Mathematics, 2014-05.

#### 3.2.5 Habilitation

[1] Ertl V.: **Constructions and Applications in $p$-adic Cohomology Theories**.
Habilitation, Universität Regensburg, Fakultät für Mathematik, 2021-04.

### 3.3 Research report

Mathematics is commonly considered to be the foundation on which science stands. It provides not only a language to describe phenomena that are observed in nature but also tools to develop new technologies.

“Ich behaupte aber, daß in jeder besonderen Naturlehre nur so viel eigentliche Wissenschaft angetroffen werden könne, als darin Mathematik anzutreffen ist.”
–Immanuel Kant: Metaphysische Anfangsgründe der Naturwissenschaft, A VIII – (1786)

I work in arithmetic geometry which traditionally is concerned with the study of integral solutions of polynomial equations as opposed to those in a field.

It is possible to study solutions of such equations purely algebraically, but not working over an (algebraically closed) field limits the effect of usual methods. However, interpreting these objects geometrically and mimicking methods from algebraic topology has been proved to be successful. For instance, one can associate algebraic data, also called “mathematical invariants”, to them, and draw conclusions about the original object by studying these algebraic structures.

An example of such a mathematical invariant is given by $p$-adic cohomology theories – which form in fact a whole forest of theories equipped to study aspects of certain geometric objects in *characteristic* $p$. I am interested in the properties, limitations and applications of different $p$-adic cohomology theories with a view on geometric applications.

### 3.3.1 Integral $p$-adic cohomology theories

In all of this section, let $k$ be a perfect field of positive characteristic $p$, and let $W(k)$ be its ring of Witt vectors. We consider varieties over $k$.

The famous Weil conjectures can be seen as a starting point for the study of $p$-adic cohomology theories. Already Weil has suggested to use a suitable cohomology theory to solve these conjectures for proper and smooth varieties over a field $k$ of characteristic $p$. For $l \neq p$, this has long been solved by Grothendieck's school using $l$-adic cohomology. The desire to fill the gap for $l = p$ motivates the search for a "good" $p$-adic theory.

The first candidate for such a theory was defined by Berthelot [3] after a suggestion due to Grothendieck in the form of crystalline cohomology $H^*_{\mathrm{cris}}(X/W(k))$. One of the motivations was to relate the seemingly unrelated objects of Dieudonné modules, $p$-adic Galois representations of local fields, and de Rham cohomology groups [33]. Conjectures based on the aforementioned correspondences were formulated by Fontaine [26], which was one of the first steps in the development of $p$-adic Hodge theory.

Crystalline cohomology can also be calculated as hypercohomology of the de Rham–Witt complex $W\Omega^\bullet$ described by Illusie in [32]. The latter one allows for concrete and intrinsic calculations while the original definition is more functorial and therefore allows more general constructions.

A drawback of crystalline cohomology is that it works well only for smooth proper schemes – for singular or open schemes, the crystalline cohomology groups are not necessarily finitely generated over $W(k)$. In this case rigid cohomology $H^i_{\mathrm{rig}}(X/K)$ introduced by Berthelot has become an important tool. However, rigid cohomology has coefficients in the quotient field $K = \mathrm{Frac}(W(k))$ of the Witt vectors, and hence it is not an integral cohomology theory.

**Overconvergent de Rham–Witt and integral Monsky–Washnitzer cohomology.** In the case of a smooth open variety, one has the problem of "controlling the sheaves on the boundary". This problem prompted Monsky and Washnitzer to impose a certain overconvergence condition on functions over the affine space in question [46]. A further step in this direction was made by Davis, Langer, and Zink when they introduced the overconvergent de Rham–Witt complex $W^\dagger\Omega^\bullet \subset W\Omega^\bullet$ as a sub-complex of the usual one in [13]. The hypercohomology of this complex is an integral cohomology theory for general smooth varieties which is rationally compatible with rigid cohomology in the quasi-projective case.

In my thesis I used this overconvergent cohomology theory to construct integral higher Chern classes and higher cycle classes for arbitrary smooth schemes over a perfect field of positive characteristic, compatible with the rigid ones in the quasi-projective case [14].

**Theorem** (Ertl)**.** *Let $X$ be a smooth $k$-variety. There is a theory of Chern classes for vector bundles and higher algebraic $K$-theory of $X$ over $k$, taking values with coefficients in the overconvergent de Rham–Witt complex*

$$c^{sc}_{ij} : K_j(X) \to H^{2j-i}(X, W^\dagger\Omega^\bullet_X).$$

*Moreover, these Chern classes are compatible with Petrequin's rigid Chern classes.*

Crystalline cohomology can take on coefficients of higher rank. The appropriate objects are so-called crystals, sheaves in the crystalline site such that the transition maps are isomorphisms. Bloch shows how the cohomology of crystals can be calculated using the de Rham–Witt complex by establishing an equivalence of categories between crystals and de Rham–Witt connections $\mathrm{Con}(X/W(k))$ [10].

Accordingly, appropriate coefficients for the overconvergent cohomology should be overconvergent connections $\mathrm{Con}^\dagger(X/W(k))$. I make this definition precise and prove that if one considers the connections that are compatible with a Frobenius map, so called $F$-de Rham–Witt connections, the overconvergent ones form a full subcategory of the convergent ones [15].

**Theorem** (Ertl)**.** *The forgetful functor*

$$j^* : F\text{-}\mathrm{Con}^\dagger(X/W(k)) \to F\text{-}\mathrm{Con}(X/W(k))$$

*is fully faithful.*

It is well-known that for smooth affine schemes (rational) Monsky–Washnitzer cohomology computes rigid cohomology. In my paper with Johannes Sprang (Universität Duisburg-Essen) [21] we could show that integrally Monsky–Washnitzer and overconvergent de Rham–Witt cohomology are compatible extending thereby results due to Davis– Langer–Zink and Davis–Zureick-Brown respectively.

**Theorem** (Ertl–Sprang)**.** *Let $\bar{A}$ be a smooth finite $k$-algebra.*

*(a) The integral Monsky–Washnitzer cohomology groups $H^i_{\mathrm{MW}}(\mathrm{Spec}\bar{A}/W(k))$ are well-defined up to unique isomorphism and are functorial in non-singular affine $k$-varieties.*

*(b) For all $i \geqslant 0$ there is a well-defined and functorial isomorphism $H^i_{\mathrm{MW}}(\bar{A}/W(k)) \xrightarrow{\simeq} H^i(W^\dagger\Omega^\bullet_{\bar{A}})$ between integral Monsky–Washnitzer and overconvergent de Rham–Witt cohomology.*

**Finiteness of integral $p$-adic cohomology.** The comparison result in [21] also means that one can make use of the computational advantages of Monsky–Washnitzer cohomology in questions about overconvergent de Rham–Witt cohomology. Finiteness questions are important and often difficult when considering cohomology theories. One can easily verify that for the affine $n$-space $\mathbf{A}^n$ over $k$, integral Monsky–Washnitzer cohomology and hence overconvergent de Rham–Witt cohomology, although integrally infinite, is finitely generated modulo torsion.

The natural question whether this remains true for more complicated examples turns out to be rather intricate. And in fact, during discussions with Atsushi Shiho (University of Tokyo), we could identify several counterexamples [19] via computations based on [34].

**Theorem** (Ertl–Shiho)**.** *For any prime number $p$ and any perfect field $k$ of characteristic $p$, there exists an affine smooth curve $X$ over $k$ such that the first integral overconvergent de Rham–Witt cohomology $H^1(X, W^\dagger\Omega^\bullet_X)$ is **not** finitely generated modulo torsion over $W(k)$.*

This negative result opens up the quest for a "good" integral $p$-adic cohomology theory once again. In a joint project with Atsushi Shiho (University of Tokyo) and Johannes Sprang (Universität Duisburg-Essen) [20] showed the following:

**Theorem** (Ertl–Shiho–Sprang)**.** *Under hypotheses of resolutions of singularities in positive characteristic there is an integral $p$-adic cohomology theory $R\Gamma_{\mathrm{rh}}(-, a^*_{\mathrm{rh}}A^\bullet)$ such that*

- *for every $k$-variety $X$, the cohomology groups $H^i(X, a^*_{\mathrm{rh}}A^\bullet)$ are finitely generated over $W(k)$,*
- *if $X/k$ has a normal crossings compactification $\overline{X}$ our cohomology is compatible with the log-crystalline cohomology of $\overline{X}$,*
- *for every $X/k$ it is rationally compatible with rigid cohomology.*

### 3.3.2 Syntomic cohomology

In this section, let $V$ be a complete discrete valuation ring of mixed characteristic $(0, p)$ with perfect residue field $k$ and fraction field $K$.

Syntomic cohomology was first introduced by Fontaine and Messing in order to prove comparison theorems between crystalline and $p$-adic étale cohomology [27]. It can be seen as a $p$-adic analogue of Deligne–Beilinson cohomology, and as such is believed to be closely related to special values of ($p$-adic) $L$-functions.

**Rigid and crystalline syntomic cohomology.** There are two constructions of syntomic cohomology. One uses (log) crystalline cohomology, and the other one uses (log) rigid cohomology In my joint work with Kazuki Yamada, we studied in particular rigid syntomic cohomology for semistable log schemes and its building blocks [22, 23]. An advantage of rigid syntomic cohomology is that it is purely $p$-adically analytic. Thus it is useful for computations of $p$-adic regulators, and should relate directly with $p$-adic $L$-functions.

A disadvantage of rigid syntomic cohomology is that the theory of log rigid cohomology often has technical difficulties because it depends a priori on the choice of local liftings. We address this point in our paper and even construct canonical logarithmic rigid complexes analogous to Besser's canonical non-logarithmic rigid complexes introduced in [7].

One of our main results is a comparison between our theory and Nekovář–Niziol's in the case of a strictly semistable scheme over $V$ with a 'nice' compactification.

**Theorem** (Ertl–Yamada)**.** *Let $X$ be a strictly semistable scheme over $V$ with normal crossing compactification, and $X_K$ its generic fibre. Then for $r \geqslant 0$ there exists a canonical quasi-isomorphism*

$$R\Gamma^{\mathrm{rig}}_{\mathrm{syn}}(X, r) \cong R\Gamma^{NN}_{\mathrm{syn}}(X_K, r).$$

*which is compatible with cup products.*

**Rigid Hyodo–Kato theory.** An important aspect of log syntomic cohomology is Hyodo–Kato theory, which was established by Hyodo and Kato [31] and allows us to construct a $(\varphi, N)$-module structure on de Rham cohomology groups of proper schemes over $V$ of semistable reduction. More precisely, they defined the crystalline Hyodo–Kato cohomology $R\Gamma^{\mathrm{cris}}_{\mathrm{HK}}(X)$ for a fine proper log scheme $X/V$ of Cartier type whose rational cohomology groups are finite dimensional vector spaces, endowed with a Frobenius-linear automorphism $\varphi$, an endomorphism $N$ satisfying $N\varphi = p\varphi N$, and a homomorphism $\Psi^{\mathrm{cris}}_{\pi} \colon H^{\mathrm{cris},i}_{\mathrm{HK}}(X) \to H^i_{\mathrm{dR}}(X_K)$ which is an isomorphism after tensoring with $K$. The latter depends on the choice of a uniformiser $\pi$ of $V$, and a change of uniformiser is encoded in a transition function involving the exponential of the monodromy.

The construction of such a Hyodo–Kato morphism is highly non-trivial. Beilinson represented crystalline Hyodo–Kato cohomology by a complex with nilpotent monodromy. What is more, his construction of the Hyodo–Kato map is independent of the choice of a uniformiser. This enabled Nekovář and Nizioł to extend syntomic cohomology to $K$-varieties. However Beilinson's Hyodo–Kato theory doesn't lend itself to explicit computations, as it is based on very abstract (crystalline) considerations. Große-Klönne proposed a rigid analytic version of the Hyodo–Kato map, based on log rigid cohomology instead of log crystalline cohomology. It provides a description of the Hyodo–Kato map in terms of $p$-adic differential forms on certain dagger spaces. However it depends on the choice of a uniformiser of $V$ and passes through several zig-zags of quasi-isomorphisms whose intermediate objects are quite complicated.

Our motivation was to establish a rigid Hyodo–Kato theory which would lend itself to explicit computation and which would be independent of the choice of a uniformiser. We accomplished this goal in [23], where we give for strictly semistable schemes $X/V$ a very intuitive construction of Hyodo–Kato cohomology $R\Gamma^{\mathrm{rig}}_{\mathrm{HK}}(X)$ and a Hyodo–Kato morphism $\Psi^{\mathrm{rig}}_{\pi,\log}$.

**Theorem** (Ertl–Yamada)**.** *Let $X$ be strictly semistable over $V$. The tuple $(R\Gamma^{\mathrm{rig}}_{\mathrm{HK}}(X), \Psi^{\mathrm{rig}}_{\pi,\log}, N, \phi)$ provides a rigid Hyodo–Kato theory in the sense that:*

*(a) The cohomology groups $H^i\left(R\Gamma^{\mathrm{rig}}_{\mathrm{HK}}(X)\right)$ are finite dimensional* $\mathrm{Frak}(W(k))$*-vector spaces.*

*(b) $\phi$ is a Frobenius-semilinear operator on $R\Gamma^{\mathrm{rig}}_{\mathrm{HK}}(X)$.*

*(c) $N$ is a nilpotent operator on $R\Gamma^{\mathrm{rig}}_{\mathrm{HK}}(X)$ such that $N\phi = p\phi N$.*

*(d) $\Psi^{\mathrm{rig}}_{\pi,\log} : R\Gamma^{\mathrm{rig}}_{\mathrm{HK}}(X) \to R\Gamma_{\mathrm{dR}}(X_k)$ is a functorial morphism which becomes an isomorphism after tensoring with $K$.*

We show that it is independent of the choice of a uniformiser and study its dependence on the choice of a branch of the $p$-adic logarithm. Moreover, we show the compatibility with the classical construction of Hyodo–Kato cohomology and the Hyodo–Kato map. We recently developed a version with compact supports of this theory [24].

**Syntomic and motivic cohomology.** In the case of a semistable $V$-scheme $X$ with a smooth special fibre, Wiesława Nizioł (Sorbonne Université) and I compute distinguished "localisation" triangle for crystalline syntomic cohomology [18]. Using as a critical new ingredient the comparison theorem between syntomic complexes and $p$-adic nearby cycles proved recently by Colmez–Nizioł, we could then prove a mixed characteristic analogue of the Beilinson–Lichtenbaum Conjecture for $p$-adic motivic cohomology.

**Theorem** (Ertl–Nizioł)**.** *We have the following natural isomorphism*

$$H^i_{\mathrm{M}}(X_{\mathrm{tr}}, \mathbf{Q}_p(r)) \xrightarrow{\sim} H^i_{\mathrm{et}}(X, \mathcal{E}(r))_{\mathbf{Q}}, \quad i \leq r,$$

*between motivic and syntomic-étale cohomology. If $X$ is proper, this yields the following natural isomorphism*

$$H^i_{\mathrm{M}}(X_{\mathrm{tr}}, \mathbf{Q}_p(r)) \xrightarrow{\sim} H^i_{\mathrm{et}}(X, \mathcal{S}(r))_{\mathbf{Q}}, \quad i \leq r,$$

*between motivic and syntomic cohomology.*

### 3.3.3 Cohomological descent

One tool that is often useful in researching cohomology theories is cohomological descent.

**An application of cohomological descent to $p$-adic Hodge theory.** An example can be found in [16] where I give a more conceptual proof of a vanishing result due to Berthelot, Esnault, and Rülling in [6]. The statement says that, for a regular proper flat scheme $X$ over a discrete valuation ring of mixed characteristic with fraction field $K$ and perfect residue field $k$, if $H^q(X_K, \mathcal{O}) = 0$ for some $q \geqslant 0$ then $H^q(X_0, W\mathcal{O})_{\mathbb{Q}} = 0$ as well. Here $X_K$ denotes the generic and $X_0$ the special fibre of $X$.

This statement is in the semistable case an application of $p$-adic Hodge theory. I realised that it is possible to use a very similar argument to obtain the more general statement of the theorem. In fact, it allows us to prove the theorem in a slightly more general case, namely for a not necessarily regular scheme but one with Du Bois singularities.

I look at the $h$-sheafification on $K$-varieties of a certain presentation of log crystalline cohomology introduced by Beilinson and use the fact that the associated cohomology groups can be seen as admissible filtered $(\varphi, N, G_K)$-modules. In particular, their Newton polygon lies above their Hodge polygon which allows us to relate de Rham and log crystalline cohomology.

One needs now descent for the $h$-topology to finish the proof. On the side of de Rham cohomology a descent result due to Huber and Jörder [29] shows that in the case of Du Bois singularities the part where the Hodge slope is $< 1$ is exactly $H^q(X_K, \mathcal{O})$, which vanishes by assumption. This means that the part of the log crystalline cohomology group where the Newton slope is $< 1$ vanishes as well. I use a descent result for Witt vector cohomology due to Bhatt and Scholze [8] to show that this is in fact $H^q(X_0, W\mathcal{O})_{\mathbb{Q}}$, which allows to conclude.

**Witt differentials in the $h$-topology.** In [29] Huber and Jörder study differential forms in the $h$-topology and their cohomology. It turns out that in characteristic 0 the $h$-sheafification of the sheaves $\Omega^n$, $n \geqslant 0$, of differential forms provides a conceptual extension of classical differential forms to study singular varieties. This approach is built on the intuition that all varieties over a field of characteristic 0 are $h$-locally smooth.

In characteristic $p > 0$ the $h$-topology encounters a number of challenges. It is possible to circumvent these difficulties to some extent with more subtle sheaf theoretic methods [30].

Keeping in mind the descent result for rational Witt vector cohomology [8], one is lead in positive characteristic to consider (rational) Witt differentials instead of usual differentials.

The hope is to extend the program described in [29] and [30] to the de Rham–Witt complex in order to obtain an equally conceptual approach to the study of singular varieties in characteristic $p$. In my article [17] with Lance E. Miller (University of Arkansas), we study the $h$-sheafification of the rational sheaf of Witt differential forms $W\Omega^n_{\mathbb{Q}}$ of degree $n \geqslant 0$. This extends the Witt differential forms to singular varieties analogous to Huber–Jörder's method described above.

To lay the base for this program, we are especially interested in descent results for rational Witt differentials. Using techniques from [30] we were able to obtain the following result:

**Theorem** (Ertl–Miller)**.** *Let $k$ be a perfect field of positive characteristic $p$. For any smooth scheme $X$ over $k$, the change of topology map induces isomorphisms*

$$W\Omega^n_{\mathbb{Q}}(X) \cong W\Omega^n_{\mathbb{Q},h}(X)$$

*for all $n \geqslant 0$.*

## 3.4 Future research projects

My research plans focus on foundational aspects and applications of $p$-adic cohomology theories. At the same time I am also interested in other aspects of mathematics, both as an inspiration and as a toolbox. These include $K$-theory, category theory, combinatorics, commutative algebra, homotopy theory, derived algebra, homological algebra and topology, to name just a few.

There are many different $p$-adic cohomology theories, which are used in different situations with different goals – they form in fact a whole forest of $p$-adic cohomologies, where it is not difficult to get lost without appropriate knowledge or a guide. There are paths between them, i.e. comparison theorems that allow us to pass between them in the appropriate situations. Working with the trees in the $p$-adic forest is often a very technical task, and some applications require a "hands-on" approach, in the sense that one has to carry out explicit calculations.

There has been for quite some time the sense that there should be an overarching theory that would allow us to unify the different $p$-adic cohomology theories, giving us a bird's eye view of the $p$-adic forest. Promising approaches are the theory of motives, which has been studied for quite some time, but also more recently prismatic cohomology [9].

One of the most beautiful and also earliest $p$-adic cohomology theories is Berthelot–Grothendieck's crystalline cohomology [3]. It is a cohomology theory with "integral" coefficients, meaning the cohomology groups are modules over a complete discrete valuation ring of mixed characteristic. However, it is well-known that the usual crystalline cohomology is only a "good" cohomology theory for proper and smooth varieties. The attribute "good" includes to be of finites type. A remedy is to work with Berthelot's rigid cohomology [4] which is of finite type even for non-smooth and open varieties. But one has to be aware of the fact that it is a rational cohomology theory, which can be a feature (for example in the context of algorithms and calculations) or a bug (for example if one wants to study $p$-torsion phenomena).

Another possibility, at least for certain singular or open varieties and schemes is to consider logarithmic structures in both the rigid and the crystalline context [31, 23]. Logarithmic $p$-adic cohomology theories encode more information which can be crucial to understand the geometric objects that we want to study. On the other hand, being able to describe the difference between the logarithmic theories and their non-logarithmic counterparts in precise mathematical terms is of advantage if one wants to calculate these invariants and hence also for the understanding of the geometric objects.

The two described dual principles, i.e. integral versus rational $p$-adic cohomology and logarithmic versus non-logarithmic $p$-adic cohomology, provide a certain tension that is a great source of interesting and important questions concerning foundational principles but also applications.

We fix some notation that will be relevant in each part of this project: Let $V$ be a complete discrete valuation ring of mixed characteristic $(0, p)$, with perfect residue field $k$, and fraction field $K$. Let $W(k)$ be the ring of Witt vectors of $k$ and denote by $K_0 = \mathrm{Frac}(W(k))$ its fraction field

### 3.4.1 Localisation triangles in rigid cohomology

In the case of a proper scheme $X/V$ of semistable reduction, Hyodo and Kato [31] introduced using crystalline techniques the eponymous cohomology $R\Gamma^{\mathrm{cris}}_{\mathrm{HK}}(X)$ together with a Frobenius $\varphi$, a monodromy operator $N$ and a Hyodo–Kato morphism $\Psi^{\mathrm{cris}}_{\pi}$. This was generalised to $K$-varieties by Beilinson [2].

Berthelot's rigid cohomology is supposed to be the kernel of the monodromy operator on the Hyodo–Kato cohomology. More precisely, the relation between the rigid cohomology and Hyodo–Kato cohomology for a flat and regular scheme $X/V$ of relative dimension $d$ can conjecturally be described by a distinguished triangle (formulated in the proper case by Flach and Morin [25])

$$R\Gamma_{\mathrm{rig}}(X_0/K_0) \xrightarrow{sp} \Big[R\Gamma^{B}_{\mathrm{HK}}(X_{\overline{K}})^{G_K} \xrightarrow{N} R\Gamma^{B}_{\mathrm{HK}}(X_{\overline{K}})^{G_K}\Big] \xrightarrow{sp'} R\Gamma^{*}_{\mathrm{rig},c}(X_0/K_0)[-2d-1] \xrightarrow{[+1]}, \qquad (1)$$

where $sp$ is a specialisation map, and $sp'$ the composition of Poincaré duality with the dual of $sp$. This triangle is related to Deligne's monodromy-weight conjecture.

In the case that $X$ has good reduction, the existence and exactness of such a triangle follows from a result by Wiesława Nizioł and myself in [18], but in the general case this conjecture is still open.

In the case of a semistable (not necessarily proper) variety $Y/k$ of dimension $d$ the definition of rigid Hyodo–Kato cohomology developed by Yamada (Keio University) and myself in [23, 24] is rather intuitive, and it is sensible to conjecture the exitence of a triangle of the form

$$R\Gamma_{\mathrm{rig}}(Y/K_0) \xrightarrow{sp} \Big[R\Gamma^{\mathrm{rig}}_{\mathrm{HK}}(Y) \xrightarrow{N} R\Gamma^{\mathrm{rig}}_{\mathrm{HK}}(Y)\Big] \xrightarrow{sp'} R\Gamma^{*}_{\mathrm{rig},c}(Y/K_0)[-2d-1] \xrightarrow{[+1]} . \qquad (2)$$

And indeed, such a result was published recently by Chiarellotto–Mazzari–Nakada [11, Thm. 3.1] in the case that $f : X \to C$ is a proper flat generically smooth morphism over $k$, where $X$ is a smooth variety of dimension $d+1$ and $C$ is a smooth curve such that for some $k$-rational point $s \in C$ the fibre $Y := X_s$ is a normal crossing divisor in $X$. Their proof adapts techniques from Chiarellotto and Tsuzuki's paper [12] on the existence and exactness of a Clemens–Schmid sequence for $p$-adic cohomology.

Our logarithmic rigid theory allows to identify

$$\left[R\Gamma_{\mathrm{HK}}^{\mathrm{rig}}(Y) \xrightarrow{N} R\Gamma_{\mathrm{HK}}^{\mathrm{rig}}(Y)\right] \cong R\Gamma_{\text{log-rig}}(Y/K_0).$$

The goal of this project is therefore to study the existence of an exact triangle

$$R\Gamma_{\mathrm{rig}}(Y/K_0) \to R\Gamma_{\text{log-rig}}(Y/K_0) \to R\Gamma^*_{\mathrm{rig},c}(Y/K_0)[-2d+1] \xrightarrow{[+1]} \tag{3}$$

as well as generalisations.

Note that there is a natural morphism

$$R\Gamma_{\mathrm{rig}}(Y/K_0) \to R\Gamma_{\text{log-rig}}(Y/K_0).$$

Thus the necessary steps are the following:

(a) Find a natural interpretation of the morphism $R\Gamma_{\text{log-rig}}(Y/K_0) \to R\Gamma^*_{\mathrm{rig},c}(Y)[-2d+1]$ which appears in the triangle.

(b) Study the exactness of the triangle in more general cases (at least the semistable case), for example with topological methods.

For this I want to follow several approaches.

**A reinterpretation of the localisation triangle of Berthelot's rigid cohomology.** Assume that $Y \subset X$ is a normal crossing divisor in a smooth (not necessarily proper) $k$-variety. Then we have the localisation triangle due to Berthelot [5, (2.3.1)]

$$R\Gamma_{Y,\mathrm{rig}}(X/K_0) \to R\Gamma_{\mathrm{rig}}(X/K_0) \to R\Gamma_{\mathrm{rig}}((X\backslash Y)/K_0) \to +$$

where $R\Gamma_{Y,\mathrm{rig}}(X/K_0)$ denotes the rigid cohomology with support in $Y$. Now one can identify $R\Gamma_{\mathrm{rig}}((X\backslash Y)/K_0)$ with the log rigid cohomology $R\Gamma_{\log-\mathrm{rig}}(X/K_0)$ where $X$ is endowed with the log structure associated to the divisor $Y$ [44]. A calculation shows further that there is a quasi-isomorphism

$$[R\Gamma_{\mathrm{rig}}(X/K_0) \to R\Gamma_{\log-\mathrm{rig}}(X/K_0)] \cong [R\Gamma_{\mathrm{rig}}(Y/K_0) \to R\Gamma_{\log-\mathrm{rig}}(Y/K_0)]$$

and hence a triangle

$$R\Gamma_{Y,\mathrm{rig}}(X/K_0) \to R\Gamma_{\mathrm{rig}}(Y/K_0) \to R\Gamma_{\log-\mathrm{rig}}(Y/K_0) \to + \tag{4}$$

Thus $R\Gamma_{Y,\mathrm{rig}}(X/K_0)$ is in fact independent of $X$, and the obvious question is, how to describe this term without a reference to $X$.

(a) A promising approach is to generalise the theory of log rigid cohomology with compact support [24] to interpret $R\Gamma_{Y,\mathrm{rig}}(X/K_0)$ in terms of certain overconvergent isocrystals.

(b) In case this approach is successful, I want to investigate whether it is possible to extend it to more general log-schemes and their differences?

Ultimately, this would provide an algebraic interpretation of a very geometric phenomenon, and hopefully simplify calculations of invariants.

**Log rigid cohomology for $K$-varieties.** Note that in the above situation by Poincaré duality $R\Gamma_{Y,\mathrm{rig}}(X/K_0) \cong R\Gamma^*_{\mathrm{rig},c}(Y/K_0)[-2d]$ thus the morphism $R\Gamma_{\text{log-rig}}(Y/K_0) \to R\Gamma^*_{\mathrm{rig},c}(Y/K_0)[-2d+1]$ in (3) can be seen as the boundary morphism of (4).

Another approach to obtain this morphism would be to pass through the compactly supported log rigid cohomology [24]. Assume that $Y$ has a compactification $\overline{Y}$ such that the horizontal divisor $D := \overline{Y}\backslash Y$ is a simple normal crossing divisor. Let $\overline{Y}^D$ be the log scheme with underlying scheme $\overline{Y}$ and log structure coming only from $D$. Then we have $R\Gamma_{\mathrm{rig},c}(Y/K_0) \cong R\Gamma_{\log-\mathrm{rig},c}(\overline{Y}^D/K_0)$ where the latter one is compactly supported towards $D$. Hence there is a canonical morphism

$$R\Gamma_{\log-\mathrm{rig},c}(\overline{Y}^D/K_0) \to R\Gamma_{\log-\mathrm{rig},c}(\overline{Y}/K_0).$$

Poincare duality induces a morphism

$$R\Gamma_{\log-\mathrm{rig}}(\overline{Y}/K_0) \cong R\Gamma_{\log-\mathrm{rig},c}(\overline{Y}/K_0)^*(-d-1)[-2d-1] \to R\Gamma_{\log-\mathrm{rig},c}(\overline{Y}^D/K_0)^*(-d-1)[-2d-1].$$

Now we might study the exactness of the triangle

$$\begin{tikzcd}
R\Gamma_{\log-\mathrm{rig}}(\overline{Y}^D/K_0) \arrow[r] \arrow[d, "\sim"] & R\Gamma_{\log-\mathrm{rig}}(\overline{Y}/K_0) \arrow[r] \arrow[d, "\sim"] & R\Gamma^*_{\log-\mathrm{rig},c}(\overline{Y}^D/K_0)(-d-1)[-2d-1] \arrow[r] \arrow[d, "\sim"] & + \\
R\Gamma_{\mathrm{rig}}(Y/K_0) \arrow[r] & R\Gamma_{\log-\mathrm{rig}}(Y/K_0) & R\Gamma^*_{\mathrm{rig},c}(YK_0)(-d-1)[-2d-1] \arrow[r] & +
\end{tikzcd}$$

Interestingly, in the lower conjectural triangle, no reference to a compactification appears, it is only in the construction of the second map that it appears.

Thus together with Kazuki Yamada we are working on a definition of log-rigid cohomology with compact support that does not need the existence of a "good" compactification. More precisely, we want to extend and refine the logarithmic rigid cohomology from [23, 24] to be able to treat $k$-varieties which are not necessarily semistable or even $K$-varieties.

For this, we want to:

(a) Formulate a suitable theory of coefficients in the category of dagger spaces.

(b) Obtain results of cohomological étale descent, using alterations on the level of dagger spaces.

(c) Obtain comparison results with the existing theory in case an integral semistable model with normal crossing compactification exists.

In addition to the localisation triangle (3), this will allow us to extend rigid Hyodo–Kato theory and hence rigid syntomic cohomology *with coefficients* to $K$-varieties. For applications to $p$-adic $L$-functions, one needs in particular an accessible theory of coefficients.

**The localisation triangle in a prismatic context.** In order to gain more insight into a certain topic, it is often beneficial to investigate it in a more general context. The prismatic cohomology introduced by Bhatt and Scholze [9] has the potential to provide a theory that allows us to step back and view the situation from a bird's eye view, as it allows via well-chosen specialisations to recover and refine different $p$-adic cohomology theories.

In this project, I investigate the objects and morphisms of the conjectural triangle (1) in prismatic terms. The hope is that the unifying character of the prismatic theory allows to make substantial progress on the conjecture, or even to extend it in the context of non-crystalline prisms.

The first part concerns a prismatic version of Hyodo–Kato theory. Ramla Abdellatif (Université de Picardie Jules Vernes) and myself have received a grant from the Bayerisch-Französischen Hochschulzentrum to investigate the monodromy operator in prismatic cohomology and relate it to interesting geometric Galois representations. Koshikawa in [36] constructed a logarithmic version of prismatic cohomology. Following his approach, it is therefore possible to obtain an object corresponding to $R\Gamma^{\mathrm{cris}}_{\mathrm{HK}}(X)$ for crystalline prisms, but also for certain more general prisms.

While this is far from completed, I would like to suggest as a follow-up project the construction of Hyodo–Kato morphism in the prismatic context, which after specialisation would provide the Hyodo–Kato morphism, from Hyodo–Kato cohomology to the de Rham cohomology. I suggest the following steps:

(a) Construction of a Hyodo–Kato morphism for crystalline prisms by transferring the approach in [23] from the rigid to the prismatic context.

(b) Extension of the construction to a Hyodo–Kato morphism for more general prisms.

(c) Construction of a prismatic syntomic cohomology using the previous results.

The second part concerns a prismatic version of rigid cohomology. As prismatic cohomology is modelled after crystalline cohomology, the same problems arise concerning open schemes. This is exacerbated by the fat that prismatically one always works with completions (in contrast to finite coefficients). In particular, the rings that one considers contain convergent series. In the rigid context however, one should rather work with overconvergent rings [4] and weak completions. This poses technical problems in the formulation of a rigid version of prismatic cohomology. There has been made some progress in this direction by Andreas Langer (not published at the moment this proposal is written).

I suggest the following steps:

(a) Integrating the log-prismatic approach of [36] with Langer's overconvergent prismatic cohomology for crystalline prisms.

(b) Extending this to an overconvergent log-prismatic cohomology for general prisms.

(c) Investigating the relation between overconvergent prismatic and overconvergent log-prismatic cohomology.

If this is successful, it is natural to investigate the relation between the overconvergent prismatic and overconvergent log-prismatic cohomology. Considering the unifying character of prismatic cohomology the hope is, that this gives us more flexible tools at hand to study a prismatic version of the triangle (3), which recovers the rigid one by specialisation.

#### 3.4.2 Relative rigid cohomology

There are roughly two approaches to construct rigid cohomology. The first one, to which Berthelot's original construction [4] belongs, uses de Rham complexes of (weak) formal lifts. It goes back to Monsky–Washnitzer [46], and was generalised by Berthelot *(loc.cit)* and Große-Klönne [28]. The other one rather uses the formalism of site studied by Ogus [40], and generalised by Shiho [42] et Le Stum [39]. The first approach, used form example in Kedlaya's algorithm, is particularly apt for explicit calculations, while the second one allows to study foundational questions. In this project I want amalgamate the two approaches to take advantage of both of them.

If $k$ is a finite field, the Zeta function of a $k$-variety encodes the number of its rational points. In cryptography it plays an important role as it allows to estimate the complexity of certain algorithms that use $p$-adic methods, mostly in elliptic curve cryptography. It can be calculated in terms of rational $p$-adic cohomology theories with Frobenius morphism. Lauder proposed a recursive method [38] to calculate certain Zeta functions: For a smooth projective family $f : X \to U \subset \mathbb{P}^1_k$, one can use a Leray spectral sequence for rigid cohomology

$$E_2^{ij} = H^j_{\mathrm{rig}}(U/K_0, R^i f_{\mathrm{rig},*}\mathscr{O}_X) \Longrightarrow H^{i+j}_{\mathrm{rig}}(X/K_0)$$

where the term on the right is Berthelot's rigid cohomology and the term on the left is a relative version [45] to reduce the dimension recursively. As mentioned above, in the context of the calculus of Zeta functions, it is important to have a Frobenius operator on the cohomology groups, and thus to describe the $E_2^{ij}$-terms as cohomology groups of overconvergent $F$-isocrytals.

**Rigid cohomology for varieties over function fields.** In this project, I propose to study the existence of a Leray spectral sequence for rigid cohomology and the properties of the $E_2$-page in more general situations. Let $S/k$ be a smooth projective geometrically irreducible variety with function field $k(S)$, and $Y/k(S)$ smooth projective. A good rigid cohomology $H^*_{\mathrm{rig}}(Y/k(S), K_0)$ for $Y$ with respect to $k(S)$ should appear in the $E_2$-page of a rigid Leray spectral sequence. Let us choose a smooth model $f : X \to U \subset S$ of $Y$ over $S$. Le Stum's overconvergent site [39] denoted by $(U/K_0)^\dagger$, provides via Grothendieck's formalism a Leray spectral sequence

$$E_2^{ij} = H^j((U/K_0)^\dagger, R^i f_*\mathscr{O}^\dagger_{X/K_0}) \Longrightarrow H^{i+j}((X/K_0)^\dagger, \mathscr{O}^\dagger_{X/K_0}) = H^{i+j}_{\mathrm{rig}}(X/K_0).$$

The problem is to interpret the coefficients $R^i f_*\mathscr{O}^\dagger_{X/K_0}$ as overconvergent $F$-isocrystals. By [1] the convergent $F$-isocrytals $R^i f_{\mathrm{conv},*}\mathscr{O}_{X/K_0}$ have a pre-image $\mathscr{E}^{\dagger,i}_{X/U}$ under the forgetful functor from overconvergent $F$-isocrystals to convergent $F$-isocrystals. Thus one might consider the cohomology groups $H^*_{\mathrm{rig}}(U/K_0, \mathscr{E}^{\dagger,i}_{X/U})$. The goal of this project is threefold:

(a) to describe $\mathscr{E}^{\dagger,i}_{X/U}$ explicitly, for example in terms of certain de Rham complexes,

(b) to identify $H^*_{\mathrm{rig}}(U/K_0, \mathscr{E}^{\dagger,i}_{X/U}) = H^*((U/K_0)^\dagger, R^i f_* \mathscr{O}^\dagger_{X/K_0})$,

(c) and to show that these cohomology groups are independent of the model $X$ of $Y$.

This will allow us to use them as a definition for $H^*_{\mathrm{rig}}(Y/k(S), K_0)$. Finally I would like to study which properties of a Weil cohomology these relative rigid cohomology groups satisfy. This would not only resolve a conjecture due to Berthelot at least in a specific case, but would also have several applications: algorithmic ones such as a generalisation of Lauder's algorithm, or theoretic ones such as in the next paragraph.

**The Tate conjecture and the Birch–Swinnerton-Dyer conjecture over function fields.** It is well-known that the Tate conjecture and the Birch–Swinnerton-Dyer conjecture are intrinsically connected and explicite relations have been established in particular cases [43]. Let $Y/k(S)$ as in the previous paragraph be in addition geometrically connected and $f : X \to U \subset S$ a smooth model over $S$. According to Grothendieck's trace formula, the Zeta function of $X$ factorises as $Z(X,t) = \prod_{n=0}^{2\dim X} L_n(X/U,t)^{(-1)^n}$, where every factor $L_n(X/U,t)$ is an Euler product associated to the $\mathbb{Q}_\ell$-sheaf $R^n\rho_*\mathbb{Q}_\ell$. If one has the Leray spectral sequence discussed above, one can also describe $L_n(X/U,t)$ in terms of $R^n\rho_*\mathcal{O}^\dagger_{X/K}$. A version of the Birch–Swinnerton-Dyer conjecture $BSD_{\mathrm{rk}}(\mathrm{Pic}^0_{Y/k(S),\mathrm{red}})$ predicts that the order of zero of $L_1(X/U,t)$ at $t = q^{\dim S}$ is equal to the rank of $\mathrm{Pic}^0_{Y/k(S),\mathrm{red}}$.

For a prime $\ell \neq p$ the $\ell$-adic Tate conjecture for a divisor $T(Y,1,\ell)$ says that the morphism

$$\mathrm{NS}(Y) \otimes \mathbb{Q}_\ell \to H^2_{\text{ét}}(Y, \mathbb{Q}_\ell(n))^{\mathrm{Gal}(\overline{k(S)}/k(S))},$$

is surjective. Using the rigid cohomology groups $H^*_{\mathrm{rig}}(Y/k(S), K)$ introduced above, one can formulate the $p$-adic analogue $T(Y,1,p)$ which says that the morphism

$$\mathrm{NS}(Y) \otimes \mathbb{Q}_p \to H^2_{\mathrm{rig}}(Y/k(S), K)^{F=p},$$

is surjective. An intermediate goal of this project is to show, by using the properties of $H^*_{\mathrm{rig}}(Y/k(S), K)$ established in the previous project, that the conjectures $T(Y,1,\ell)$ for any $\ell$ are equivalent.

If $T(Y,1,\ell)$ is true, one knows [41] that $BSD_{\mathrm{rk}}(\mathrm{Pic}^0_{Y/k(S),\mathrm{red}})$ is equivalent to the identity

$$V_\ell Ш_{k(S)}(\mathrm{Pic}^0_{Y/k(S),\mathrm{red}}) = 0$$

for $\ell \neq p$, where $Ш_{k(S)}(\mathrm{Pic}^0_{Y/k(S),\mathrm{red}})$ denotes the Tate–Shafarevich group and $V_\ell$ indicates its rational Tate module. The ultimate goal of this project is to show the same result for $\ell = p$ by using rigid analytic methods, in particular the relative rigid cohomology groups $H^*_{\mathrm{rig}}(Y/k(S), K)$ of the first paragraph and the rigid Leray spectral sequence.

### 3.4.3 Integral $p$-adic cohomology theories

Note that in the previous part rational cohomology theories are considered. However as mentioned above, inverting $p$ loses information. It is an important quest to find a "good" integral $p$-adic cohomology theory, particularly to answer questions of $p$-torsion. History shows that this is a very subtle problem.

First one should clarify what we mean by a "good" integral $p$-adic cohomology theory. Reasonable properties are:

- For every variety over a perfect field of characteristic $p > 0$, the cohomology groups should be $W(k)$-modules of finite type.
- In the case of a smooth variety with a normal crossing compactification one should recover logarithmic crystalline cohomology.
- One should obtain a rational comparison with rigid cohomology.

In [20] we obtain for any $k$-variety $X/k$ an integral cohomology theory $H^*_{\mathrm{cdh}}(X/W(k))$ which satisfies these conditions, but under strong conditions on resolutions of singularities in positive characteristic. While this is not ideal yet, it gives at least a good indication of what might be possible. Thus it is reasonable to study on the one hand additional structures on our theory and on the other hand to explore alternative methods to construct a "good" integral $p$-adic cohomology theory.

Concerning the first point, it is natural to hope for a weight filtration and a slope filtration on or integral $p$-adic cohomology theory. It is well known that such structures play an important role in $p$-adic Hodge theory. I suggest to study the existence and interaction of these two filtrations on our cohomology. This is related to a project with Moritz Kerz within project A03 in the SFB 1085 "Higher Invariants" , where we would like to study the Nygaard filtration in the framework of the $p$-adic homotopy developed in [35].

With Atsushi Shiho, I plan to extend our theory to a version with compact support and obtain Poincaré duality. Moreover, it would be enlightening to transfer our topological methods to the prismatic context.

Concerning the second point, one lead that Atsushi Shiho and I follow is the construction of a relative version of our cohomology theory. The steps are:

(a) Formulation of a relative version $H^*_{\mathrm{cdh}}(Y/X, W(k))$ of the integral $p$-adic cohomology [20] for a morphism of $k$-varieties $f : Y \to X$ in terms of relative de Rham–Witt [37] complexes together with a Gauß–Manin connection.

(b) Reformulation of this in terms of crystalline higher direct images.

This should lead to a Leray spectral sequence of the following form:

$$E_2^{ij} = H^j(X/W(k), R^i f_{\mathrm{cdh},*}\mathscr{O}_Y) \Longrightarrow H^{i+j}_{\mathrm{cdh}}(Y/W(k))$$

which is an integral version of the rigid Leray spectral sequence appearing in the previous part. The hope is, that such a spectral sequence will allow to reduce the use of resolution of singularities in positive characteristic to small dimensions.

# References


[1] Ambrosi E., *Specialization of Néron-Severi groups in positive characteristic.* Preprint, arXiv:1810.06481, (2021).

[2] Beilinson A.: *p-adic periods and derived de Rham cohomology.* J. of the AMS, vol. 25, no. 3, pp. 715–738, (2012).

[3] Berthelot P.: *Cohomologie cristalline des schémas de caractéristique* $p > 0$. Lecture Notes in Math., vol. 127, Springer-Verlag, (1974).

[4] Berthelot P.: *Géométrie rigide et cohomologie des variétés algébriques de caractéristique p.* Mémoires de la S.M.F. 2e série, vol. 23, pp. R3–3 and 7–32, (1986).

[5] Berthelot P.: *Finitude et pureté cohomologique en cohomologie rigide (with an appendix in English by de Jong A.J.).* Invent. Math. vol. 128, pp. 329–377, (1997).

[6] Berthelot P., Esnault H., Rülling K.: *Rational points over finite fields for regular models of algebraic varieties of Hodge type* $\geqslant 1$. Ann. Mat., vol. 176, pp. 412–508, (2012).

[7] Besser A.: *Syntomic regulators and p-adic integration I: rigid syntomic regulators.* Israel J. of Math., vol. 120, pp. 291–334, (2000).

[8] Bhatt B., Scholze P.: *Projectivity of the Witt vector affine Grassmannian.* Invent. Math., vol. 209, no. 2, pp. 329–423, (2017).

[9] Bhatt B.; Scholze P.: *Prisms and Prismatic Cohomology.* Preprint arXiv:1905.08229, (2019).

[10] Bloch S.: *Crystals and de Rham-Witt connections.* J. de l' Institut de Math. de Jussieu, vol. 3, pp. 315–326, (2004).

[11] Chiarellotto B., Mazzari N., Nakada Y.: *A Conjecture of Flach and Morin* Preprint, arXiv:230309839, (2023).

[12] Chiarellotto B.;Tsuzuki N.: *Clemens–Schmid exact sequence in characteristic $p$.* Math. Ann. vol. 358(3-4), pp. 971–1004, (2014).

[13] Davis C., Langer A., Zink T.: *Overconvergent de Rham-Witt Cohomology.* Ann. Sci. Ec. Norm. Supér. (4), vol. 44, no. 2, pp. 197-262 (2011).

[14] Ertl V.: *Overconvergent Chern Classes and Higher Cycle Classes.* Dissertation, University of Utah, College of Science, Department of Mathematics, 2014-05.

[15] Ertl V.: *Full faithfulness for overconvergent $F$-de Rham–Witt connections.* C. R. Math., vol. 354, no. 7, pp. 653–658, (2016).

[16] Ertl V.: *A new proof of a vanishing result due to Berthelot, Esnault, and Rülling.* Journal of Number Theory, DOI: 10.1016/j.jnt.2019.03.022 (2019).

[17] Ertl V.; Miller L.E.: *Witt differentials in the $h$-topology.* Journal of Pure and Applied Algebra, vol. 223, no. 12, pp. 5285–5309, (2019).

[18] Ertl V.; Nizioł W.: *Syntomic cohomology and $p$-adic motivic cohomology.* Algebraic Geometry, vol. 6, pp. 1–32, (2019).

[19] Ertl V.; Shiho A.: *On infiniteness of integral overconvergent de Rham–Witt cohomology modulo torsion.* Tohoku Mathematical Journal, vol. 72, no. 3, pp. 395–410, (2020).

[20] Ertl V., Shiho A. et Sprang J.: *Integral $p$-adic cohomology theories for open and singular varieties.*
Preprint arXiv:2105.11009, (2021).

[21] Ertl V.; Sprang J.: *Integral comparison of Monsky-Washnitzer and overconvergent de Rham-Witt cohomology.* Proceedings of the AMS, ser. B, vol. 5, pp. 64–72, (2018).

[22] Ertl V.; Yamada K.: *Comparison between rigid syntomic and crystalline syntomic cohomology for strictly semistable log schemes with boundary.* Rend. Sem. Mat. Univ. Padova, vol. 145, pp. 213–291, (2021).

[23] Ertl V.; Yamada K.: *Rigid analytic reconstruction of Hyodo–Kato theory.* Preprint arXiv:1907.10964, (2019).

[24] Ertl V.; Yamada K.: *Poincaré duality for rigid analytic Hyodo–Kato theory.* Preprint arXiv:2009.09160, (2020).

[25] Flach M.; Morin B.: *Weil-étale cohomology and Zeta-values of proper regular arithmetic schemes.* Documenta Mathematica, vol. 23, pp. 1425–1560, (2018).

[26] Fontaine J.-M.: *Groupes $p$-divisible sure les corps locaux.* Astérisque, pp. 47–48, SMF, (1977).

[27] Fontaine J.-M.; Messing W.: *$p$-adic periods and étale cohomology.* Contemp. Math., vol. 87, pp. 176–207, (1987).

[28] Grosse-Klönne E.: *De Rham cohomology for rigid spaces.* Math. Z.,vol. 247, no. 2, pp. 223–240, (2004).

[29] Huber A., Jörder C.: *Differential forms in the $h$-topology.* Algebr. Geom., vol. 1, no. 4, pp. 449–478, (2014).

[30] Huber A., Kebekus S., Kelly S.: *Differential forms in positive characteristic avoiding resolution of singularities.* Bulletin de la SMF, vol. 45, fas. 2, pp. 305–343, (2017).

[31] Hyodo O.; Kato K.: *Semi-stable reduction and crystalline cohomology with logarithmic poles.* Astérisque vol. 223, pp. 221–268, (1994).

[32] Illusie L.: *Complex de deRham-Witt et cohomologie cristalline.* Ann. Sci. Ec. Norm. Supér. (4), vol. 12, no. 4, pp. 501–661 (1979).

[33] Illusie L.: *Grothendieck at Pisa: crystals and Barsotti-Tate groups.* Talk at the colloquium di Giorgi, Pisa, (2013).

[34] Kedlaya K.S.: *Counting points on hyperelliptic curves using Monsky–Washnitzer cohomology.* J. Ramanujan Math. Soc., vol. 16, no. 4, pp. 323–338, (2001).

[35] Kerz, M.; Saito, S.; Tamme, G. *K-theory of non-Archimedean rings. I.* Documenta Mathematica, vol. 24, pp. 1365–1411, (2019).

[36] Koshikawa, T.: *Logarithmic Prismatic Cohomology I.* Preprint `arXiv:2007.14037`, (2020).

[37] Langer A.; Zink T.: *De Rham–Witt cohomology for a proper and smooth morphism.* Journal of the Institute of Mathematics of Jussieu, vol. 3, no. 2, pp. 231–314, (2007).

[38] Lauder A.: *A recursive method for computing zeta functions of varieties.* LMS J. Comp. Math., vol. 9, pp. 222–267, (2006).

[39] Le Stum B.: *The Overconvergent site.* Mémoire de la SMF, vol. 127, (2011).

[40] Ogus A.: *The Convergent Topos in Characteristic p.* In: Cartier, P., Illusie, L., Katz, N.M., Laumon, G., Manin, Y.I., Ribet, K.A. (eds) *The Grothendieck Festschrift.* Modern Birkhäuser Classics, vol. 88, Birkhäuser, (2007).

[41] Qin Y.: *Comparison of different Tate conjectures.* Preprint, `arXiv:2012.01337`, (2020).

[42] Shiho A.: *Crystalline fundamental groups I – Isocrystals on log crystalline site and log convergent site.* J. Math. Sci. Univ. Tokyo, vol. 7, no. 4, pp. 509–656, (2000).

[43] Tate J.T.: *Conjectures on algebraic cycles in l-adic cohomology.* In *Motives* (Seattle, WA, 1991), pp. 71–83, (1994).

[44] Tsuzuki N.: *On the Gysin isomorphism of rigid cohomology.* Hiroshima Math. J. vol. 29, pp. 479–527, (1999).

[45] Tsuzuki, N.: *On base change theorem and coherence in rigid cohomology.* Documenta Mathematica, Extra Volume Kato, pp. 891–918, (2003).

[46] van der Put M.: *The cohomology of Monsky and Washnitzer.* Mémoires de la SMF, Nouvelle Série vol. 23, pp. 33–59, (1986).

## 3.5 Received funding and awards

**Heisenberg Programm**

Personal grant awarded by the DFG
Project: "Approaches to and applications of $p$-adic cohomology theories"
Accepted into the program: October 2023

**Grant of the Bavarian-French University Centre**

Research cooperation with Ramla Abdellatif, Université de Picardie Jules Verne (France)
Project: "The monodromy operator in prismatic cohomology"
Amount: 6235 €
January 2022 – December 2023

**ARSP grant**
Personal grant within the "Academic Research Sabbatical Program"
Awarded by the Universität Regensburg
Project: "Aspects of $p$-adic cohomologies" Amount: 29400 €
April 2019 – September 2019

**JSPS postdoctoral fellowship**
Personal grant for research in Japan at Keio University
Selected by the Alexander von Humboldt Foundation
Project: "Logarithmic rigid cohomology theories" Amount: 4744000 ¥
Host: Prof. K. Bannai
August 2017 – July 2018

**DFG collaborative research center SFB 1085: "Higher Invariants – Interactions between Arithmetic Geometry and Global Analysis"**
Collective grant within the Fakultät für Mathematik, Universität Regensburg
Awarded by the DFG
Role: PI for the third funding period
2022 – 2026

**Scholarship for habilitation**
Personal grant
Awarded by the Bavarian state
Project: "Research in $p$-adic cohomologies"
Amount: 13000 €
March 2017 – July 2017

**Travel grant for female junior scientists**
Personal grant
Awarded by the Bavarian state
Project: "Research in $p$-adic cohomologies"
Amount: 3306 €
May 2017 – July 2017

**DFG research training group GRK 1692: "Curvature, Cycles, and Cohomology"**
Collective grant within the Fakultät für Mathematik, Universität Regensburg
Awarded by the DFG
Role: member, associate
2010 – 2019

**ERASMUS-scholarship** Academic year (Master 1) in France
Amount: 3600 €
September 2006 – August 2007

## 3.6 Research talks

### 3.6.1 Research talks at conferences

**October 2023**
A Conference in Arithmetic Algebraic Geometry in Memory of Jan Nekovář, IHES, Bures-sur-Yvette (France)
Talk: **Conjectures on $L$-functions for Varieties Over Function Fields and Their Relations**

**September 2022**
Conference in honour of Bruno Chiarellotto "Around $p$-adic Cohomologies", Padova (Italy)
Talk: **Poincaré duality in log rigid cohomology**

**May 2022**

BIRS-CMO Workshop "Advances in Mixed Characteristic Commutative Algebra & Geometric Connections", Casa Matemática Oaxaca (Mexico)
Talk: **Integral p-adic cohomology for open and singular varieties**

**December 2020**

Conference "Tropical Geometry, Berkovich Spaces, Arithmetic D-Modules and $p$-adic Local Systems", Imperial College London (UK)
Talk: **Poincaré duality for rigid Hyodo–Kato theory**

**September 2019**

Conference in honour of Bernard Le Stum "Over and around sites in characteristic $p$", Padova (Italy)
Talk: **A rigid analytic approach to Hyodo–Kato theory**

**May 2019**

London–Paris Number Theory Seminar, Kings College London (UK)
Talk: **Crystalline and rigid syntomic cohomology for strictly semistable schemes**

**February 2019**

Oberwolfach Workshop "Singularities and Homological Aspects of Commutative Algebra", Oberwolfach (Germany)
Talk: **Witt differentials and the h-topology**

**November 2018**

Oberwolfach Seminar "Syntomic Cohomology and $p$-adic Hodge Theory", Oberwolfach (Germany)
Talk: **The rigid Hyodo–Kato morphism**

**July 2018**

Sendai–Hiroshima Number Theory Conference, Sendai (Japan)
Talk: **A vanishing result due to Berthelot, Esnault, and Rülling**

**April 2018**

Arkansas Spring Lecture Series, "Old and New themes in $p$-adic Cohomology", Fayetteville (USA)
Talk: **Crystalline and rigid syntomic cohomology for strictly semistable schemes**

**February 2018**

Project Research Conference in Hakone, Hakone (Japan)
Talk: **A vanishing result due to Berthelot, Esnault, and Rülling**

**November 2017**

Conference "Algebraic Geometry with Fancy Coefficients", Caen (France)
Talk: **Syntomic cohomology and $p$-adic motivic cohomology**

**October 2017**

BIRS Workshop "$p$-adic Cohomology and Arithmetic Appliations", Banff (Canada)
Talk: **Integral Monsky–Washnitzer and overconvergent de Rham–Witt cohomology**

**May 2017**

Workshop on Arithmetic Geometry, Hakodate (Japan)
Talk: **Integral Monsky–Washnitzer and overconvergent de Rham–Witt cohomology**

**September 2016**

Conference "Differential forms in algebraic geometry", Freiburg (Germany)
Talk: **Witt differentials and the $h$-topology**

**September 2015**

Workshop "$p$-adic Hodge theory and Iwasawa theory", Bielefeld (Germany)
Talk: **Rigid and arithmetic syntomic cohomology**

**July 2014**

Oberwolfach Arbeitstagung "Algebraic Number Theory", Oberwolfach (Germany)
Talk: **Overconvergent de Rham–Witt connections**

### 3.6.2 Research talks in seminars

**February 2024**
Université de Caen Normandie, Number Theory Seminar, Caen (France)
Talk: **Conjetures sur les fonctions $L$ sur les corps de fonctions**

**January 2024**
University of Warwick, Number Theory Seminar, Coventry (UK)
Talk: **Conjectures on $L$-functions for Varieties Over Function Fields and Their Relations**

**January 2024**
Universität Münster, Number Theory Seminar, Münster (Germany)
Talk: **Conjectures on $L$-functions for Varieties Over Function Fields and Their Relations**

**December 2023**
Università degli Studi di Milano, Arithmetic Geometry Seminar, Milano (Italy)
Talk: **Conjectures on $L$-functions for Varieties Over Function Fields and Their Relations**

**November 2023**
University of Warsaw, Algebraic Geometry Seminar, Warszawa (Poland)
Talk: **An introduction to $p$-adic cohomology theories**

**November 2023**
IMPAN Algebraic Geometry Seminar (impanga), Warszawa (Poland)
Talk: **Conjectures on $L$-functions for Varieties Over Function Fields and Their Relations**

**March 2023**
Université de Bordeaux, Number Theory Seminar, Bordeaux (France)
Talk: **Constructions rigides en théorie de Hodge $p$-adique**

**January 2023**
Universität Heidelberg, GAUS-Seminar, Heidelberg (Germany)
Talk: **Integral $p$-adic cohomology for open and singular varieties**

**December 2022**
Université de Picardie Jules Verne, Number Theory Seminar, Amiens (France)
Talk: **Constructions rigides pour la cohomologie de Hyodo-Kato**

**March 2022**
Université de Caen Normandie, Number Theory Seminar, Caen (France)
Talk: **Le noyaux de la monodromie en cohomologie p-adique – une approche rigide analytique**

**March 2022**
Université de Strasbourg, Arithmetic Geometry seminar, Strasbourg (France)
Talk: **Integral $p$-adic cohomology for open and singular varieties**

**February 2022**
Universität Wuppertal, Algebra Oberseminar, Wuppertal (Germany)
Talk: **Integral $p$-adic cohomology for open and singular varieties**

**October 2021**
Université Rennes 1, Arithmetic Geometry Seminar, Rennes (France)
Talk: **The kernel of the monodromy in $p$-adic cohomology - a rigid analytic approach**

**March 2021**
Université de Caen, Number Theory Seminar, Caen (France)
Talk: **Constructions rigides pour la cohomologie de Hyodo-Kato**

**February 2020**
Isaac Newton Institute, seminar of the program “$K$-theory, algebraic cycles and motivic homotopy theory”, Cambridge (UK)
Talk: **A rigid analytic approach to Hyodo–Kato theory**

**July 2018**
Keio University, Algebra Seminar, Yokohama (Japan)
Talk: **A vanishing result due to Berthelot, Esnault, and Rülling**

**November 2017**
Sophia University, colloque du Départment de Mathématiques, Tokyo (Japan)
Talk: **Logarithmic and non-logarithmic $p$-adic cohomology theories**

**May 2017**
Keio University, Algebra Seminar, Yokohama (Japan)
Talk: **Syntomic cohomology and $p$-adic motivic cohomology**

**November 2014**
Albert–Ludwigs Universität Freiburg, Algebra Seminar, Freiburg (Germany)
Talk: **Overconvergent de Rham–Witt connections**

**October 2014**
Universität Regensburg, colloquium GRK 1692 “Curvature, Cycles, and Cohomology”, Regensburg (Germany)
Talk: **Overconvergent de Rham–Witt connections**

**October 2014**
Universität Bielefeld, Number Theory Seminar, Bielefeld (Germany)
Talk: **Overconvergent de Rham–Witt connections**

**January 2014**
Universität Regensburg, Number Theory Seminar, Regensbrug (Germany)
Talk: **Overconvergent Chern classes**

**January 2014**
Universität Regensburg, colloquium GRK 1692 “Curvature, Cycles, and Cohomology”, Regensburg (Germany)
Talk: **Overconvergent de Rham–Witt complex and applications**

**January 2014**
Humboldt Universität Berlin, Number Theory Seminar, Berlin (Germany)
Talk: **Overconvergent Chern classes**

**January 2014**
California Institute of Technology, Number Theory Seminar, Pasadena (USA)
Talk: **Overconvergent Chern classes**

**May 2013**
UC San Diego, Number Theory Seminar, La Jolla (USA)
Talk: **Overconvergent Chern classes**

## 3.7 Poster presentations

**January 2018**
UK-Japan Winter School “Galois representations and automorphic forms”, King’s College London (UK)
Poster presentation: **Syntomic and motivic cohomology**

**February 2017**
Third Meeting for Young Women in Mathematics "Cohomological Methods in Geometry", Freiburg (Germany)
Poster presentation: **Witt differentials and the $h$-topology**

**May 2015**
Second Meeting for Young Women in Mathematics "Cohomological Methods in Geometry", Freiburg (Germany)
Poster presentation: **Overconvergent $F$-connections**

**June 2014**
Evaluation of the DFG research training group GRK 1692 "Curvature, Cycles, and Cohomology", Universität Regensburg, Regensburg (Germany)
Poster presentation: **Overconvergent $F$-connections**

# 4 Pedagogical merits

## 4.1 Teaching profile

Throughout my career, I had the opportunity to teach in different settings and mentor students at different points of their career. In my previous position as "Akademische Rätin auf Zeit", I tought 5 hours per week, which corresponds to 145 hours per year.

As a graduate student instructor at the University of Utah, I taught mostly undergraduate classes for students whose major was outside of mathematics. At the Universität Regensburg, I had the opportunity to teach classes on all levels, undergraduate and graduate.

This includes service lectures, such as Analysis for Physicists, or a class that prepares future secondary school teachers for the state exam in Algebra.

In Utah as well as in Regensburg I organised seminars: undergraduate seminars, where I had to guide students and teach them how to acquire knowledge and how to present a talk, but also graduate seminars and research seminars.

As a former international student and researcher in the US, France and Japan, it is particularly important for me that international students and colleagues feel welcomed and are integrated into the community.

## 4.2 Teaching experience

### 4.2.1 University lectures

***Preparation class for the Algebra exam for secondary school teachers***
Universität Regensburg, summer term 2023
Language: German
Level: Master equivalent
Number of participants: 33

***Elementary Geometry for middle school teachers***
Universität Regensburg, winter term 2022/23
Language: German
Level: Bachelor equivalent
Number of participants: 54

**Assistant for Dr. Strunk's preparation class for the Algebra exam**
Universität Regensburg, summer term 2022
Language: German
Level: Master equivalent
Number of participants: 43

**Assistant for Dr. Bowden's preparation class for the Analysis exam**
Universität Regensburg, summer term 2022

Language: German
Level: Master equivalent
Number of participants: 43

***Preparation class for the Algebra exam for secondary school teachers***
Universität Regensburg, winter term 2021/22
Language: German
Level: Master equivalent
Number of participants: 45

**Assistant for Dr. Ludewig's class "Analysis for physics"**
Universität Regensburg, winter term 2021/22
Language: German
Level: Bachelor
Number of participants: 30

**Assistant for Prof. Naumann's class "Linear Algebra 1"**
Universität Regensburg, winter term 2020/21
Language: German
Level: Bachelor
Number of participants: 250

***Algebraic groups***
Universität Regensburg, summer term 2020
Language: English/German
Level: Master
Number of participants: 4
Hours per week: 4

***Analysis for physics***
Universität Regensburg, winter term 2019/20
Language: German
Level: Bachelor
Number of participants: 35
Hours per week: 6

***Preparation class for the Algebra exam for secondary school teachers***
Universität Regensburg, winter term 2018/19
Language: German
Level: Master equivalent
Number of participants: 40
Hours per week: 6

**Exercise session for Prof. Naumann's class "Algebra"**
Universität Regensburg, winter term 2016/17
Language: German
Level: Bachelor
Number of participants: 45
Hours per week: 2

***Preparation class for the Algebra exam for secondary school teachers***
Universität Regensburg, winter term 2016/17
Language: German
Level: Master equivalent
Number of participants: 40
Hours per week: 4

**Assistant for Prof. Naumann's class "Linear Algebra 2"**
Universität Regensburg, summer term 2016

Language: German
Level: Bachelor Number of participants: 150

***Preparation class for the Algebra exam for secondary school teachers***
Universität Regensburg, summer term 2016
Language: German
Level: Master equivalent
Number of participants: 40
Hours per week: 4

***Lectures on the de Rham–Witt complex***
Universität Regensburg, winter 2015/16
Language: English
Level: PhD students
Number of participants: 4
Hours per week: 4

**Teaching assistant for Prof. van Opstall's class "Math 1010: Intermediate Algebra"**
University of Utah, spring term 2014
Language: English
Level: Bachelor
Number of participants: 100

***Math 1210: Calculus I***
University of Utah, fall term 2013
Language: English
Level: Bachelor
Number of participants: 24
Hours per week: 4

***Math 1100: Quantitative Analysis***
University of Utah, spring term 2011
Language: English
Level: Bachelor
Number of participants: 18
Hours per week: 4

***Math 1030: Introduction to Quantitative Reasoning***
University of Utah , fall term 2010
Language: English
Level: Bachelor
Number of participants: 22
Hours per week: 4

**Teaching Assistant for Prof. de Fernex' class "Math 3210: Foundations of Analysis I"**
University of Utah, spring term 2010
Language: English
Level: Bachelor
Number of participants: 30

**Teaching Assistant for Prof. Lodh's class "Math 3220: Foundations of Analysis II"**
University of Utah, spring term 2010
Language: English
Level: Bachelor
Number of participants: 30

**Teaching Assistant for Prof. Schmitt's class "Math 3210: Foundations of Analysis I"**
University of Utah, fall term 2009
Language: English

Level: Bachelor
Number of participants: 30

**Teaching Assistant for Prof. Treiberg's class "Math 3220: Foundations of Analysis II"**
University of Utah, fall term 2009
Language: English
Level: Bachelor
Number of participants: 30

**Exercise session for Prof. Morel's class "Algebra II"**
Ludwig–Maximilians Universität München, summer term 2009
Language: German
Level: Bachelor
Number of participants: 15
Hours per week: 2

**Exercise session for Prof. Morel's class "Algebra I"**
Ludwig–Maximilians Universität München, winter term 2008/09
Language: German
Level: Bachelor
Number of participants: 15
Hours per week: 2

**Exercise session for Prof. Morel's class "Linear Algebra II"l**
Ludwig–Maximilians Universität München, summer term 2008
Language: German
Level: Bachelor
Number of participants: 15
Hours per week: 2

**Exercise session for Prof. Morel's class "Linear Algebra I"**
Ludwig–Maximilians Universität München, winter term 2007/08
Language: German
Level: Bachelor
Number of participants: 15
Hours per week: 2

#### 4.2.2 Mini courses

***Different syntomic cohomology theories***
Sophia University, Tokyo (Japan)
Lecture and discussion day, June 2017
Language: English
Level: post-docs
Number of participants: 3

***The de Rham–Witt complex***
Albert-Ludwigs-Universität Freiburg (Germany)
three-day lecture series, January 2015
Language: German
Level: PhD students, post-docs
Number of participants: 15

#### 4.2.3 Student seminars

**Mathemagika: mathematics and magic**
Universität Regensburg, summer term 2021
Language: German

Level: Bachelor
Number of participants: 12

**Crystallographic groups**
Universität Regensburg, summer term 2016
Language: German
Level: Bachelor
Number of participants: 13

**Finite flat group schemes**
Universität Regensburg, summer term 2014
Language: German
Level: Bachelor
Number of participants: 3

## 4.3 Supervision

**Research interships**
2021, François Trinh : "$p$-divisible gropus"
2021, Zhenghui Li : "The Weil conjecture – Deligne's proof"

**Bachelor theses**
2020 – 2021, Joshua Lappat : "Lie algebras and the universal envelopping algebra"
2020 – 2021, Lukas Wolfseher : "Abelian categories and $\mathfrak{G}$-modules"

**PhD theses**
2022 – . . . Andrea Panontin, cosupervised with Moritz Kerz

## 4.4 Pedagogical material

My teaching material can be found on my teaching web site. Most of the courses that I taught at the Universität Regensburg are in German.

A few highlights are the following:

- Lecture notes for **Elementary geometry**, including a study guide
- Lecture notes for **Algebraic groups**, taught as an inquiry based learning course
- Lecture notes for **Algebra**
- Lecture notes for **Lectures on the de Rham–Witt complex**
- Development of a concept to prepare future secondary school teachers for the state exam including additional learning material

## 4.5 Theoretical knowledge

I completed the following pedagogical classes

- **Personal Impression – intensive workshop**
  Universität Regensburg/ Carpe Verba, February 2016
- **Instructor Training Workshop for International Graduate Students**
  University of Utah, August 2009
- **Instructor Training Workshop for Mathematics**
  University of Utah, August 2009

## 4.6 Thoughts on mathematics at the university

As early as in the first century, Seneca deplores in one of his letters to Lucillius the fact that the Roman philosophy school only teaches empty knowledge unrelated to life's challenges criticising thereby the educational system of his time. He ends his statement with the words: "Non vitae, sed scholae discimus." – "We do not learn for life but for school." [*Epistulae morales ad Lucillium* 106, 11-12]

Better known is the inversion of this expression – "Non scholae, sed vitae discimus." – meant as a request for schools to provide knowledge and skills which will benefit the students in life. How can we realise this in mathematics?

The task of an instructor of mathematics is to transmit mathematical knowledge and skills to their students which they need in their work and career. It seems obvious that mathematics plays an important role for scientists, engineers, economists, just to name a few.

However, one could ask, whether mathematics has an intrinsic value.

Naturally, "mathematics" is a very wide area, and when we are teaching, one can find oneself in very different situations, depending on the subject, the students' background and their goals, and many other factors. Is there a common denominator?

To answer this question, let us think about a different question, namely: What does it actually mean to "do math"? Of course this can differ from person to person, from situation to situation. But it often means that one tries to understand a concept, a situation, a fact as deeply as possible. One starts from a certain knowledge, maybe realises, that some information is missing, or that there is a problem. Then one sets out to solve this problem, to follow the logic and see where it leads without compromising the truth. Sometimes, one can follow a recipe, or modify an existing approach, but often a totally new strategy is needed. One might struggle, wrestle with the problem, run in the wrong direction, until finally after so-and-so-many attempts, one understands the nature of the problem just a little bit more, and is one step closer to a solution. The satisfaction, when one finally finds a solution, or sees something razor sharp in one's mind, is beyond words. The process can be humbling and empowering at the same time.

In other words, mathematics can teach us to recognise a problem, to endure the anxiety of not understanding or not knowing, to tackle the problem with the tools that we have, to acquire the necessary knowledge that we lack, to persevere until we find a solution, or until we can say we did everything in our power. During this process, we can learn a lot not only about our environment, but also about ourselves.

Often we go beyond what we thought lies in our ability, just by following the path of logic one step at a time. And we only realise afterwards what we have accomplished. Other times, we learn to recognise our limitations, and that seeking help is not a shame. We learn to formulate our thoughts precisely, to find the right resources, or to talk to colleagues and classmates.

With time, we gain more experience and knowledge, hone our creativity, learn to interact with other people, but also with problems and abstract concepts. This can be seen as a (very) broad definition of mathematical literacy. Having such abilities can help a person to play an active role in society in general.

Independently of the concrete mathematical subject and its level, learning mathematics is a chance to cultivate the mentioned abilities. As instructors we have the privilege to assist the students in this effort. We have to recognize where the students are when the come into our class and give them the right input to bring them to the next level.

But what is the "right input"? Obviously, the students have to gain knowledge, learn methods and facts which have been accumulated by mathematicians throughout human history. But gradually, they have to emancipate and find their own problems and solutions. Thus it is important for an instructor to find the balance between guiding the students and giving them enough freedom to grow.

I keep the following principles in mind when I design my classes, seminars and assignments. I aim to require complex and non-algorithmic thinking, meaning that a predictable approach is not explicitly suggested by the task, or a worked out example. I want to encourage the students to explore and understand the nature of mathematical concepts, processes, and relationships. The exercises should require them to access relevant knowledge and experiences and to make appropriate use of them while working through the task. The solution should require considerable cognitive effort and may involve some level of anxiety on the part of the student because of the unpredictable nature of the solution process.

There are many different aspects of teaching and each has its own challenges. But being able to lead talented students into the exciting world of mathematics, or to accompany them on a part of their path is very rewarding. Moreover, the dialogue with students often helps me to see things in a different light, and even benefits me in my research. It provides the opportunity for both sides to grow together, both mathematically and as persons.

## 4.7 Description of selected experiences

### 4.7.1 Classes for students outside of a classical career

At the University of Utah I taught several evening/night classes which were ideal for students with other obligations during the day, for example a class called "**Quantitative analysis**". It was mostly aimed at business students who learned concepts such as differentiation, maximisation and minimisation of functions and integration. Several of the studetns had already been working for several years, or had been taking care of their family, and decided to come back to school and pursue a degree. As a consequence, they were particularly motivated, but also faced many challenges, such as a tight schedule, or financial problems. I learned that a key for them to be successful was to allow them to work more autonomously, but without compromising the content and quality of the course. While the subjects of these classes seem elementary, and in many countries are taught at the high school level, one needs to understand that they were a challenge for many of the students, either because they didn't follow a scientific path during their secondary education or because they entered university later in life.

### 4.7.2 Service lectures

In Regensburg and in Utah, I gave lectures in collaboration with other departments, to teach students techniques and knowledge that they would need in a specific career. For example, during the winter term 2019/20 I had the privilege to teach the class "**Analysis III for Physicists**". This class encompasses real and complex analysis, as well as differential equations. The students should not only learn the statements and how to apply them, but also should understand the concepts and ideas behind, including proofs.

In this context I also coordinated the oral mathematics exams that the physics students have to take during their bachelor.

### 4.7.3 Master/PhD classes

At the University of Regensburg, I was given the opportunity to teach some advanced level classes. In the winter term 2015/16, I gave a lecture on the **de Rham–Witt complex** for doctoral students. As this is not a routinely taught class I wrote notes for this lecture which are accessible on my website. The class was based partly on Illusie's famous article, but also covered some more recent rests, such as the big de Rham–Witt complex by Hesselholt–Madsen.

### 4.7.4 Teachers education

The state of Bavaria conducts central written exams which all future secondary school (Gymnasium) teachers need to pass. One of them covers algebra (group theory, ring theory, field theory (Galois theory)), and basic number theory. While they will in all likelihood not use these subjects in their work life as teachers – unless they have an exceptionally gifted student – the idea behind this is to give them a broad scientific background knowledge and deeper appreciation for mathematics. The University of Regensburg offers a class which aims to assist the students in their preparation for this exam. I was partly or fully responsible for this preparation class for several terms. I developed a concept for this class with the goal to improve the results in the state exam. On my web page I provide resources for them, such as summaries of important subjects, a study guide, or problem sheets, that they can use during class or for their personal studies. My personal wish for the students is that the pressure of getting a good result in the exam doesn't overshadow the beauty of mathematics and joy of problem solving.

### 4.7.5 Interactive classes

The last two and the present term are maybe some of the most challenging during my career, due to the coronavirus pandemic As the usual course of action at universities is disrupted, it can be a challenge to make interactions among students and between students and professors/instructors possible, We have now the responsibility and opportunity to make use of the technical possibilities. During the summer term 2020 I gave a course on algebraic groups. I took this as an opportunity to use a different approach based on the method of Moore. I prepared lecture notes where only the definitions and statements where given, and proofs where spared out. The students were encouraged to look at the material beforehand, so that we could discuss the proofs during the lectures. There were several active participants which ensured exciting discussions. The students got a feeling what it means to collaborate in mathematical problem solving, but also that often one has to dig deep to fully understand a problem or concept.

#### 4.7.6 Seminar organisation

In Regensburg and in Utah I organised several hot topic seminars and student seminars. For example, I co-organised a hot topic seminar on **algebraic de Rham cohomology** within the framework of the "DFG collaborative research centre SFB 1085" **Higher Invariants – Interactions between Arithmetic Geometry and Global Analysis**. I also organised "pros-seminars", seminars for students at the beginning of their scientific career, typically within their first year of studies, where they are supposed to learn how to give mathematical (or more generally scientific) talks. Take as an example a proseminar on crystallographic groups. As an instructor, I guide the students in their preparation, but also give them feedback afterwards, to help them give talks of a good quality.

# 5 Administration, service, organisation

## 5.1 Organisation of hot topic seminars

**Summer term 2023**
Universität Regensburg, Regensburg (Germany)
Oberseminar **A new construction of the de Rham–Witt complex after Bhatt–Lurie–Mathew**
with M. Kerz

**Winter term 2016/17**
Universität Regensburg, Regensburg (Germany)
Higher Invariants Oberseminar **Algebraic de Rham cohomology**
with F. Strunk et G. Tamme

**Summer term 2015**
Universität Regensburg, Regensburg (Germany)
Oberseminar **Weil's conjecture on Tamagawa numbers**

**Winter term 2014/15**
Universität Regensburg, Regensburg (Germany)
Oberseminar **Non-archimedean geometry**

**Spring term 2013**
University of Utah, Salt Lake City (USA)
Seminar **$p$-adic Deformation of algebraic cycle classes**

**Fall term 2012**
University of Utah, Salt Lake City (USA)
Seminar **Étale cohomology**
with L. E. Miller

**Spring 2010**
University of Utah, Salt Lake City (USA)
Seminar **The theorem of Čerednik and Drinfeld**
with R. Lodh

## 5.2 Assignments as reviewer

- Journal of Number Theory
- Proceedings of the London Mathematical Society
- Nagoya Mathematical Journal
- Proceedings volumes of the Simons Symposia
- Mémoirs of the AMS
- Compositio Mathematica
- Bulletin de la Société Mathématique de France
- Algebra & Number Theory
- International Mathematics Research Notices

## 5.3 Outreach/mentoring

- Discussion group for gifted female students in mathematics `https://homepages.uni-regensburg.de/~erv10962/Outreach/talent.htm`
  April 2020 – September 2023
- Information workshop for master students `https://homepages.uni-regensburg.de/~erv10962/Outreach/infodiss.htm`
  December 2020 – January 2021
- Open day for secondary school students, November 2014, (Universität Regensburg)
- Contributions to `http://www.suri-joshi.jp/` a resource for girls in mathematics: 1, 2, 3

## 5.4 Administration

- Representative for the junior members of the department in the **faculty council**
  October 2021 – September 2022
- Representative for the junior members of the department in the **hiring committee** for a W3 professorship
  October 2021 – June 2022
- PI in the DFG collaborative research centre **SFB 1085 "Higher Invariants"**
  December 2022 – November 2026
- Note taker for oral exams
  May 2016 – September 2023
- Organisation and coordination of the oral mathematics exams for physics students
  winter 2019/20
  winter 2021/22
- Responsible for the online material for Prof. N. Naumann's class "Linear Algebra 1"
  winter 2020/21

**Data:** 21/01/2024 **Luogo:** Varsavia